Time dependent integral method for Maxwell's system

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Abstract. *We solve the problem of diffraction of an electromagnetic wave by a perfectly conducting body using a boundary integral method in time-domain directly. In this work, we extended to the three-dimensional case the method developed by A. PUJOLS [6] in 2D using spaces coupling time and space introduced by I. TERRASSE [8]. By studying the associated frequency domain problem, we prove the continuity of the associated areal operator. We obtain results of stability and convergence in this time functional framework. The discret approximation of the variational formulation leads to a stable marching-in-time scheme.*

1.Introduction.– The time dependent integral method was applied by HA DUONG [4] in 1987 to solve the equation of waves in dimension 3D+1. He also defined a functional framework used by E. BECACHE [2] in elastic waves . It is in this framework, that A. PUJOLS [6] proposed in 1991 a variational formulation for Maxwell's system and implemented the associated scheme in 2D+1. In 1993, I. TERRASSE [8] introduced new spaces coupling time and space and resolved numerically Maxwell equations by Lagrange's multiplyings. We decided to study the formulation given in [6] in 3D in this new framework, and to implement it directly.

Then, we consider a tridimensional object Ω^-, with regular bounded surface Γ and exterior normal $\vec{n}$, lighted by an incident wave (E^i, H^i) that hits the scatterer at $t = 0$.

The scattered field (E, H) satisfies in $\Omega^+ = \mathbb{R}^3 \backslash \overline{\Omega^-}$ the Maxwell equations:

$$(P^+)\quad \begin{cases} \overrightarrow{curl} E + \mu_0 \partial_t H = \vec{0} & in\ \mathbb{R}^t \times \Omega^+ \\ \overrightarrow{curl} H - \varepsilon_0 \partial_t E = \vec{0} & in\ \mathbb{R}^t \times \Omega^+ \\ div E = 0 = div H & in\ \mathbb{R}^t \times \Omega^+ \\ E \wedge \vec{n}_{|\Gamma} = \vec{g} & on\ \mathbb{R}^t \times \Gamma \\ E(t,.) = 0 = H(t,.) & for\ t<0 \end{cases}$$

The tangent vector $\vec{g}$ is given by the incident electric field

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$$\vec{g} = -E^i \wedge \vec{n}_{|\Gamma} \quad on\ \mathbb{R}^t \times \Gamma$$

The constants μ_o and ε_o are respectively the magnetic permeability and the electric permittivity . We consider so an interior problem defined by extending the field (E, H) into the interior domain Ω^-:

$$(P^-)\quad \begin{cases} \vec{curl E} + \mu_o \partial_t H = \vec{0} \ in\ \mathbb{R}^t \times \Omega^- \\ \vec{curl H} - \varepsilon_o \partial_t E = \vec{0} \ in\ \mathbb{R}^t \times \Omega^- \\ div E = 0 = div H \quad in\ \mathbb{R}^t \times \Omega^- \\ E \wedge \vec{n}_{|\Gamma} = \vec{g} \qquad on\ \mathbb{R}^t \times \Gamma \\ E(t,.) = 0 = H(t,.) \quad for\ t<0 \end{cases}$$

2.Representation of (E, H) by retarded potentials.– By using the classical integral representation, the solution of $(P^{\pm})$ can be represented by the retarded simple layer L of densities $\vec{j}$ and ρ:

$$(1)\qquad E = -\mu_0 L \partial_t \vec{j} - \vec{grad}_x L\rho \ in\ \mathbb{R}^+ \times (\Omega^+ \cup \Omega^-)$$

$$(2)\qquad H = \vec{curl} L \vec{j} \ in\ \mathbb{R}^+ \times (\Omega^+ \cup \Omega^-)$$

where L is the operator:

$$Lp(t,x) = \int_\Gamma \frac{p(t-|x-y|_c, y)}{4\pi |x-y|} d\Gamma(y) \ for\ t \in \mathbb{R}^+ \ and\ x \in \mathbb{R}^3$$

where $|x-y|_c$ designs the ratio $\frac{|x-y|}{c}$ $with\ c^2 = \frac{1}{\mu_0 \varepsilon_0}$, and the surface current and charge $\vec{j}$ and ρ are the jumps:

$$\vec{j}(t,x) = \vec{n} \wedge H^+_{|\Gamma} - \vec{n} \wedge H^-_{|\Gamma} \ \forall (t,x) \in \mathbb{R}^+ \times \Gamma$$

$$\rho(t,x) = \vec{n}.E^+_{|\Gamma} - \vec{n}.E^-_{|\Gamma} \ \forall (t,x) \in \mathbb{R}^+ \times \Gamma$$

They are connected by the equation of conservation of charge:

$$(3)\qquad div_\Gamma \vec{j} + \varepsilon_0 \partial_t \rho = 0 \ on\ \mathbb{R}^+ \times \Gamma$$

The mathematical study will be easier if we use an intermediate unknown $\vec{\varphi}$ defined by the jump:

$$\vec{\varphi} = \vec{curl} E^- \wedge \vec{n}_{|\Gamma} - \vec{curl} E^+ \wedge \vec{n}_{|\Gamma} \ on\ \mathbb{R}^+ \times \Gamma$$

The representations become:

$$(4)\qquad E(t,x) = L\vec{\varphi}(t,x) - c^2 \vec{grad}_x L\left(\int_0^t \int_0^s div_\Gamma \vec{\varphi}(\sigma, x) d\sigma ds\right) \ in\ \mathbb{R}^+ \times (\Omega^+ \cup \Omega^-)$$

(5) $$H(t,x) = -\frac{1}{\mu_o}\, curl_x L\left(\int_0^t \vec{\varphi}(s,x)ds \right) \quad in\ \mathbb{R}^+ \times (\Omega^+ \cup \Omega^-)$$

Applying the boundary condition on the tangentiel field, we obtain the integral equation:

(6) $$H\vec{\varphi}(t,x) = \vec{n} \wedge \vec{g}(t,x) \quad \forall\, (t,x) \in \mathbb{R}^+ \times \Gamma$$

with

- the operator H given by:

$$H\vec{\varphi}(t,x) = \Pi_{\Gamma x} \int_\Gamma \frac{1}{4\pi|x-y|}\, \vec{\varphi}(t-|x-y|_c), y)\, dy - c^2 \overrightarrow{grad}_{\Gamma x} \int_\Gamma \frac{1}{4\pi|x-y|} \int_0^{t-|x-y|_c} \int_0^s div_\Gamma \vec{\varphi}(\sigma,y) d\sigma dy$$

- and the maps:

$$\Pi_{\Gamma x} f(x) = -\vec{n}(x) \wedge (\vec{n}(x) \wedge f_{|\Gamma}) \overrightarrow{grad}_{\Gamma x} f(x) = -\vec{n}(x) \wedge (\vec{n}(x) \wedge \overrightarrow{grad}_x f_{|\Gamma})$$

We can see that the resolution of the equation (6) leads to determinate the field (E,H) in $\Omega^+ \cup \Omega^-$ thanks to relations (4)(5).

3.Functional framework.– Before introducing the functional framework, we shall do some recalls about the Fourier-Laplace transform. Let E be an Hilbert space, we note $\mathcal{D}'_+(E)$ the set of E-valued distributions, and $\mathcal{S}'(E)$ the set of E-valued tempered distributions with support in $\mathbb{R}^+$. For real σ, $\sigma>0$, we can define:

$$LT(\sigma,E) = \{ T \in \mathcal{D}'_+(E),\ e^{-\sigma t} T \in \mathcal{S}'_+(E) \}$$

and the Fourier-Laplace transform $\hat{T}$ of T:

$$\hat{T}(\omega) = \mathcal{F}(e^{-\sigma t} T)(\eta) = \int_{-\infty}^{+\infty} e^{i\omega t} T(t) dt$$

where $\mathcal{F}$ is the usual Fourier transform and the frequency $\omega = \eta + i\sigma$.

We are ready to introduce the spaces presented by I. TERRASSE in [8]. For $s \in \mathbb{R}$, and $\sigma \in \mathbb{R}$, we define:

$$H^s_\sigma(\mathbb{R}^+, H^\sigma(\Omega)) = \{ f \in LT(\sigma, H^\sigma(\Omega)),\ \int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s} \|\hat{f}(\omega)\|^2_{\sigma,\omega,\Omega} d\omega < +\infty \}$$

which is an Hilbert space with the norm:

$$\|f\|^2_{s,\sigma,H^\sigma(\Omega)} = \frac{1}{2\pi} \int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s} \|\hat{f}(\omega)\|^2_{\sigma,\omega,\Omega} d\omega$$

with:

$$\forall\, y \in H^\sigma(\Omega),\ \|\hat{f}\|^2_{\sigma,\omega,\Omega} = \frac{1}{|\omega|} \|\hat{f}^\omega(y)\|^2_{H^\sigma(|\omega|\Omega)} \quad where \quad \forall\, y \in H^\sigma(|\omega|\Omega),\ \hat{f}^\omega(y) = \frac{1}{|\omega|} \hat{f}\left(\frac{y}{\omega}\right)$$

This definition was extended on the surface Γ. Before, we have to introduce the spaces:

$$H^\sigma(div,\Gamma) = \{ f \in H^\sigma(\Gamma) : f.\vec{n} = 0,\ div_\Gamma f \in H^\sigma(\Gamma) \}$$

$$H^{\sigma}(curl,\Gamma) = \{ f \in H^{\sigma}(\Gamma) \colon f.\vec{n} = 0,\ curl_{\Gamma} f \in H^{\sigma}(\Gamma) \}$$

Then, for $s \in \mathbb{R}$ and $\sigma \in \mathbb{R}$

$$H^{s}_{\sigma}(\mathbb{R}^{+},H^{\sigma}(div,\Gamma)) = \{ f \in LT(\sigma,H^{\sigma}(div,\Gamma)), \int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s} \|\hat{f}(\omega)\|^{2}_{\sigma,\omega,div_{\Gamma}} d\omega < +\infty \},$$

with the associated norm:

$$\|f\|^{2}_{s,\sigma,H^{\sigma}(div,\Gamma)} = \frac{1}{2\pi}\int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s} \|\hat{f}(\omega)\|^{2}_{\sigma,\omega,div_{\Gamma}} d\omega$$

and

$$\forall\, y \in H^{\sigma}(div,|\Gamma|), \quad \|f\|^{2}_{\sigma,\omega,\Gamma} = \|\hat{f}^{\omega}(y)\|^{2}_{H^{\sigma}(div,|\omega|\Gamma|)}$$

(resp $H^{s}_{\sigma}(\mathbb{R}^{+},H^{\sigma}(curl,\Gamma))$)

These norms are equivalent to the usual norms and we have the

Proposition 1. *For all* $|\omega| \geq \sigma$, *and* $\sigma \geq 0$:

$$C(\sigma)^{-(\sigma+1)} \|\hat{f}\|_{\sigma,\omega,div_{\Gamma}} \leq \|\hat{f}\|_{H^{\sigma}(div,\Gamma)} \leq C(\sigma)^{(\sigma+1)} |\omega|^{(\sigma+1)} \|\hat{f}\|_{\sigma,\omega,div_{\Gamma}}$$

$$C(\sigma)^{-(\sigma+1)} |\omega|^{-\sigma} \|\hat{f}\|_{-\sigma,\omega,div_{\Gamma}} \leq \|\hat{f}\|_{H^{-\sigma}(div,\Gamma)} \leq C(\sigma)^{(\sigma+1)} |\omega|\, \|\hat{f}\|_{-\sigma,\omega,div_{\Gamma}}$$

where $C(\sigma) = sup(\frac{1}{\sigma},1)$

We have the same result for $H^{\sigma}(curl,\Gamma)$ by replacing in *Proposition 1.*, *div* by *curl*.

4.Variational problem.– Adopting HA-DUONG's approach, we study the associated harmonic problem to deduce properties of H by using Fourier-Laplace transform in time.

Harmonic problem.

Therefore, the integral equation (6) becomes:

(7) $$H_{\omega}\hat{\varphi}(x) = \vec{n} \wedge \hat{g}(x) \quad \forall\, x \in \Gamma$$

where

- $\hat{\varphi}$ is the jump of $\overrightarrow{curl}\hat{E} \wedge \vec{n}$ through Γ.
- $\hat{g} = -\hat{E} \wedge \vec{n}_{|\Gamma}$
- H_{ω} is the operator defined by:

(8) $$H_{\omega}\hat{\varphi}(x) = \overrightarrow{grad}_{\Gamma x} \int_{\Gamma} \frac{c^{2}}{\omega^{2}} \frac{e^{i\omega|x-y|_{c}}}{4\pi|x-y|} div_{\Gamma}\hat{\varphi}(y)\, dy + \Pi_{\Gamma x} \int_{\Gamma} \frac{e^{i\omega|x-y|_{c}}}{4\pi|x-y|} \hat{\varphi}(y)\, dy$$

We can prove the

Proposition 2.

For $\omega \in \mathbb{C}$, $Im(\omega)=\sigma>0$, H_ω *is an isomorphism from* $H^{-1/2}(div,\Gamma)$ *into* $H^{-1/2}(curl,\Gamma)$ *and satisfies* :

$$\|H_\omega \hat{\varphi}\|_{-1/2,\omega,rot_\Gamma} \leq \frac{C}{\sigma}\|\hat{\varphi}\|_{-1/2,\omega,div_\Gamma} \qquad \forall \hat{\varphi} \in H^{-1/2}(div,\Gamma)$$

The associated sesquilinear form :

(9) $$b_\omega(\hat{\varphi},\hat{\psi}) = \langle \varphi, -i\omega H_\omega \hat{\psi}\rangle \quad \forall\ \psi, \varphi \in H^{-1/2}(div,\Gamma)$$

satisfies the coercivity condition [8]

(10) $$Re\ (b_\omega(\hat{\varphi},-i\omega\hat{\varphi})) \geq C(\Gamma)\frac{\sigma}{|\omega|}\|\hat{\varphi}\|^2_{-1/2,\omega,div_\Gamma} \qquad \forall \hat{\varphi} \in H^{-1/2}(div,\Gamma)$$

Hence, the harmonic problem (7) can be solved by the standard variational method.

Time-dependent problem.

Applying the inverse Fourier-Laplace transform to the solution of harmonic problem, the time dependent problem (6) is well-posed : for $\vec{g}$ given in $H^s_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$, and $s \in \mathbb{R}$, there exists a unique solution $\vec{\varphi} \in H^{s-2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$.
Parseval formula applied to (9) leads to the space-time variational formulation of time dependent problem :

(11) $$b(\vec{\varphi},\vec{\psi}) = \int_0^\infty e^{-2\sigma t}\langle \vec{n} \wedge \vec{g}, \partial_t \vec{\psi}\rangle\, dt \quad \forall\ \vec{\psi} \in H^{1-s}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$$

where

$$b(\vec{\varphi},\vec{\psi}) = \int_0^\infty e^{-2\sigma t}\langle H\vec{\varphi}, \partial_t \vec{\psi}\rangle\, dt.$$

The braket denotes the product of duality $H^s_\sigma(\mathbb{R}^+,H^{-1/2}(curl,\Gamma))\times H^{-s}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$.
We remark that the continuity of H and coercivity relation (10) implies the continuity of b into $H^s_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))\times H^{1-s}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$ and the coercivity relation :

$$b(\vec{\varphi},\vec{\varphi}) \geq C(\Gamma)\,\sigma\,\|\vec{\varphi}\|^2_{-1/2,\sigma,H^{-1/2}(div,\Gamma)} \quad \forall\ \vec{\varphi} \in H^{1/2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$$

5.Approximation of the variational problem.– We want now to discretize the variational form (11) using finite elements. We first make a space approximation.We construct an approximate surface Γ_h of Γ composed by regular triangles Γ_i. Hence, we consider the edge element family of RAVIART-THOMAS [7] divergence conforming space V_h consisting of polynomials of degree one. P^{-1} denotes the inverse map of $(P_{|\Gamma_h}: \Gamma_h \to \Gamma)$. We can also define the space:

$$\widetilde{V}_h = \{\ \widetilde{\varphi} = \varphi \circ P^{-1}\ ;\ \varphi \in V_h\}$$

Then, the unknown $\vec{\varphi}$ is represented by an expansion of basis function $\vec{\varphi}_j$ *for* j=1,*Ne* of $\widetilde{V}_h$ as:

$$\vec{\varphi}(t,y) \approx \vec{\varphi}_h(t,y) = \sum_{j=1}^{Ne} \alpha_j(t)\,\vec{\varphi}_j(y)$$

where $\alpha_j \in H^s_\sigma(\mathbb{R}^+,\mathbb{R})$, $s\in \mathbb{R}$.

We choose the test function as:

$$\vec{\psi}(t,x) \approx \vec{\psi}_h(t,x) = \beta_i(t)\,\vec{\varphi}_i(x)$$

where $\beta_i \in H^{1-s}_\sigma(\mathbb{R}^+,\mathbb{R})$,.

The discret problem consists in finding α_j for j=1,Ne such that:

$$(12)\ \sum_{j=1}^{Ne}\int_0^\infty e^{-2\sigma t}\partial_t\beta_i(t)\int_\Gamma\int_\Gamma\{\ K^{(1)}_{ij}(x,y)\alpha_j(t-|x-y|_c) + c^2K^{(2)}_{ij}(x,y)\int_0^{t-|x-y|_c}\int_0^r\alpha_j(s')ds'dr\ \}$$
$$)d\Gamma(x)\ d\Gamma(y)dt$$

$$=\int_0^\infty e^{-2\sigma t}\int_\Gamma \vec{n}\wedge\vec{g}_h(t,x)\ .\ \vec{\varphi}_i(x)d\Gamma(x)\ dt$$

where $\vec{g}_h$ is an approximation of $\vec{g}$ in $H^s_\sigma(\mathbb{R}^+,\tilde{V}_h)$ and $K^{(1)}_{ij}$ and $K^{(2)}_{ij}$ are defined by:

$$K^{(1)}_{ij}(x,y) = \frac{\vec{\varphi}_i(x)\ .\ \vec{\varphi}_j(y)}{4\pi|x-y|}\quad et\ K^{(2)}_{ij}(x,y) = \frac{div_\Gamma\vec{\varphi}_i(x)\,div_\Gamma\vec{\varphi}_j(y)}{4\pi|x-y|}$$

In a second step, the positive time axis is divided into subintervals $I_k = [t_k,t_{k+1}]$ of length Δt. The function of $H^s_\sigma(\mathbb{R}^+,\mathbb{R})$ is approximated by those of the subspace $H^m_\sigma(\Delta t,\mathbb{R})$, $m \in \mathbb{N}$, composed of polynomials of degree $m\geq s$ in each time interval I_k.

Thanks to the continuity and the coercivity of b, there is a unique solution $\vec{\varphi}^t_h$ *in* $H^{m_1}_\sigma(\Delta t,\tilde{V}_h)$, $m_1 \in \mathbb{N}$ and $m_1 \geq s$ of the discret problem :

$$(12)\,b(\vec{\varphi}^t_h,\vec{\psi}^t_h) = \int_0^\infty e^{-2\sigma t}<\vec{n}\wedge\vec{g}^t_h,\partial_t\vec{\psi}^t_h>dt\quad\forall\ \vec{\psi}^t_h\in H^{m_2}_\sigma(\Delta t,\tilde{V}_h),\ m_2\in\mathbb{N}\ and\ m_2\geq 1-s$$

where

$$b(\vec{\varphi}^t_h,\vec{\psi}^t_h) = \int_0^\infty e^{-2\sigma t}<H\vec{\varphi}^t_h,\partial_t\vec{\psi}^t_h>dt.$$

Before choosing time approximations, we present results of stability and convergence.

6. Stability and convergence.– These results are obtained for an exact surface. If Γ is approached by a surface Γ_h, we can follow Nedelec's idea [5].We want to use the continuity and the relation of coercivity of b, then we take s=1/2 and we obtain the result of stability.

Theorem 3 *Let* $\vec{g}^t_h$ *be a consistant approximation of* $\vec{g}$ *in* $H^{3/2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$, *then there exists* $c>0$ *such that* :

$$\|\vec{\varphi}^t_h\|_{-1/2,\sigma,H^{-1/2}(div,\Gamma)} \leq C\ when\ h\to 0\ and\ \Delta t\to 0.$$

We can prove the convergence of the method by adapting the Lemma of interpolation of [4] and estimates of [3] on the functional framework introduce in [8] thanks to the proposition 1.

Lemme 4 *For all* $\vec{p}^{\,t}_h \in H^{m_2}(\Delta t, \widetilde{V}_h)$, $m_2 > 0$, *we have:*

$$\| \vec{p}^{\,t}_h \|_{1/2,\sigma,H^{-1/2}(div,\Gamma)} \leq C\, h^{-1/2} \Delta t^{-2} \| \vec{p}^{\,t}_h \|_{-1/2,\sigma,H^{-1/2}(div,\Gamma)}$$

Then, we can deduce the theorem of convergence:

***Theorem* 5** *We suppose that* $\vec{\varphi} \in H^{m_1+2}_\sigma(\mathbb{R}^+, H^{m_1}(div,\Gamma)) \cap H^{m_2+2}_\sigma(\mathbb{R}^+, H)$, *with* $m_1 > 1$, $m_2 > 2$ *We denote* $H = H^{-1/2}(div,\Gamma) \cap L^2(\Gamma)$. *Then for all* $\varepsilon \in]0, \frac{1}{2}]$:

$$\| \vec{\varphi} - \vec{\varphi}^{\,t}_h \|_{-1/2,\sigma,H^{-1/2}(div,\Gamma)} \leq C\,(\ \| \vec{g} - \vec{g}^{\,t}_h \|_{3/2,\sigma,H^{-1/2}(div,\Gamma)}$$

$$+ \frac{h^{m_1-\varepsilon-1/2}}{\Delta t^2} \| \vec{\varphi} \|_{m_1+2,\sigma,H^{m_1}(div,\Gamma)} + \frac{\Delta t^{m_2-2}}{h^{1/2}} \| \vec{\varphi} \|_{m_2+2,\sigma,H}\ )$$

7.Numerical results.–

Now we return to the approached surface Γ_h.To obtain a stable and convergent scheme, we should choose an unknown in $H^4(\Delta t, V_h)$, that is unrealizable! So, one tests an approximation by constant polynomials *i.e.* s=0 and the stability and convergence will be tested numerically, with two sorts of incident waves: a sinusoidal wave and an impulsion. We take $\sigma = 0$.

Scheme $\mathbb{P}^t_0 \times \mathbb{P}^t_1$.

Firstly, let us consider the simplest choice of approximations.The approximate functions of α_j and β_i are taken in $H^0(\Delta t, \mathbb{R})$ and in $H^1(\Delta t, \mathbb{R})$ respectively:

$$\alpha_j(t) \approx \sum_{m \geq 1} \chi^m(t) \alpha^m_j \quad for\ j = 1, Ne \qquad \chi^m(t) = \begin{cases} 1 & if\ t \in [t_m, t_{m+1}[\\ 0 & elsewhere \end{cases}$$

$$\beta_i(t) \approx \beta^l(t) \alpha^l_i \quad for\ i = 1, Ne \qquad \beta^l(t) = \begin{cases} 0 & if\ t \leq t_{l-1} \\ t - t_l & if\ t \in [t_{l-1}, t_l[\\ \Delta t & if\ t \geq t_l \end{cases}$$

With a simple substitution into (12) and some additional manipulations, we obtain the matrix form:

$$(14) \qquad \begin{cases} D^0 \vec{X}^1 = \vec{SM}^1 \\ D^0 \vec{X}^l = -\sum_{p=1}^{l-1} D^{l-p} \vec{X}^p + \vec{SM}^l \quad for\ l \geq 2 \end{cases}$$

with

$$D1^p_{ij} = \int_\Gamma \int_\Gamma \{ K^{(1)}_{ij}(x,y) \int_0^{\Delta t} \chi^{l-p}(s + t_{l-1} - |x-y|_c) ds$$

$$+ c^2 K^{(2)}_{ij}(x,y) \int_0^{\Delta t} \int_0^{s + t_{l-1} - |x-y|_c} \int_0^r \chi^{l-p}(s') ds' dr\, ds \ \} \ d\Gamma(x)\ d\Gamma(y)$$

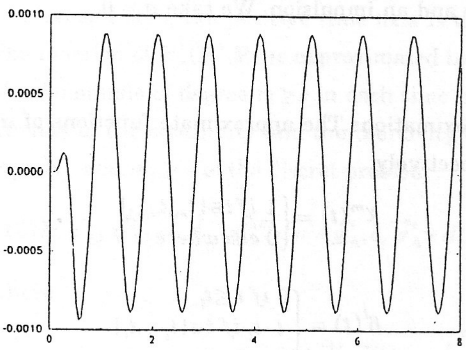
The discret problem is a quasi-explicit marching-in-time scheme: a single inversion of the matrix D^0 is required. We have tested this scheme on a sphere of radius 1m approached by 80 triangles. We represent the solution computed for different values of CFL= $c\frac{\Delta t}{\Delta x}$ at the lighted point of the object. For CFL=0.3; 0.5 or 0.8, one can observe a quick blowing up. So, we decided to take another time approximation: the unknown and the test function are both chosen in $H^0_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$.

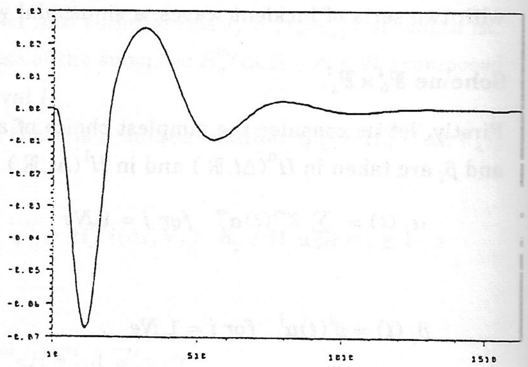
Scheme $\mathbb{P}_0' \times \mathbb{P}_0'$.

The functions α_j and β_i are approximated by:

$$\alpha_j(t) \approx \sum_{m\geq 1} \chi^m(t)\alpha_j^m \; for\; j = 1, Ne \qquad\qquad \beta_i(t) \approx \chi^l(t)\alpha_i^l \; for\; i = 1, Ne$$

We obtain a similar matrix system as (14) and now the scheme is stable. The two figures present results for a sinusoidal wave and an impulsion respectively for CFL=0.5 and the frequency F=50MHz. We take always 10 grid points/wavelengh. Increasing the number of triangles, frequency can be taken higher. Results have been valued using the theorem of limited amplitude.





7.Conclusion.– In this paper, we have solved Maxwell's system for conducting obstacles by an integral method based of the representation of the electric field by retarded potentials on the surface Γ. By using Fourier-Laplace transform, we obtain a well-posed variational problem and the continuity of the associated sesquilinear form. The stability and convergence are proved but require a to high order approximation. So, we consider a weaker approximation and obtain a numerical stability. The scheme is constructive and is directly solved.Therefore, we can conclude that our scheme is perfectly robust and stable.

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