BOUNDARY INTEGRAL EQUATION METHOD IN TIME DOMAIN FOR MAXWELL'S SYSTEM

Alain Bachelot, *Bordeaux I University*
Agens Pujols, *C.E.A./C.E.S.T.A.*

Abstract. *This paper deals with the calculation of time–dependent electromagnetic field near a perfectly conducting body. Our approach for obtaining the scattered wave is based on a boundary integral equation method applied to acoustics by A. Bamberger and T. Ha Duong. By studying the associated frequency domain problem, we prove this integral equation corresponds to a space–time coercive variational problem. The discrete approximation of the variational formulation leads to a stable marching–in–time scheme. We present numerical computations in 2D+1.*

1. Scattering problem for a conducting body . We consider the scattering of an incident electromagnetic wave (E^i , H^i) by a tridimensional object Ω_- with regular bounded surface Γ. In the exterior domain , Ω_+ , the scattered field satisfies the Maxwell equations (P_+) and the solution (E , H) of (P_+) can be extended in Ω_- so that the interior problem (P_-) is also satisfied :

$$(P_\pm)\begin{cases} -\partial_t E + curl\, H = 0\,,\ \partial_t H + curl\, E = 0\,, in\ \mathbb{R}_t \times \Omega_\pm \\ div\, E = div\, H = 0\,, in\ \mathbb{R}_t \times \Omega_\pm \\ \vec{n} \wedge E = -\vec{n} \wedge E^i = c\ \ , on\ \mathbb{R}_t \times \Gamma\ c=0\,, t \le 0, \end{cases}$$

where $\vec{n}$ denotes the unit normal to the surface Γ, pointing into Ω_+ . Then it is well-known that the solution (E , H) of (P_+) and (P_-) can be written in terms of *retarded potentials* :

$$E(t\,,x) = -\frac{1}{4\,\pi}\int_\Gamma \frac{\partial_t j(\tau,y)}{|x-y|}\,d\sigma_y - \frac{1}{4\,\pi}grad\int_\Gamma \frac{q(\tau,y)}{|x-y|}\,d\sigma_y \qquad (1)$$

$$H(t\,,x\,) = \frac{1}{4\,\pi}rot\int_\Gamma \frac{j(\tau,y)}{|x-y|}\,d\sigma_y\ ,\quad t \in \mathbb{R}^+\ and\ x \notin \Gamma \qquad (2)$$

where the *retarded time* $\tau = t - |x-y|$. The surface currents and charges j and q are the jumps of $-\vec{n} \wedge H$ and $-\vec{n}.E$ through Γ, respectively. They are connected by the equation of charge conservation :

$$\partial_t q + div_\Gamma j = 0\,. \qquad (3)$$

Hence, surface current j determines E :

$$E(t,x)=-\frac{1}{4\pi}\int_\Gamma \frac{\partial_t j(\tau,y)}{|x-y|}d\sigma_y+\frac{1}{4\pi}grad\int_\Gamma\int_0^\tau \frac{div_\Gamma j(s,y)}{|x-y|}ds\,d\sigma_y,$$

and the boundary condition becomes for $x\in\Gamma$

$$-\vec{n}\wedge\{\vec{n}\wedge\frac{1}{4\pi}\int_\Gamma \frac{\partial_t j(\tau,y)}{|x-y|}d\sigma_y\}-\frac{1}{4\pi}grad_\Gamma\int_\Gamma\int_0^\tau \frac{div_\Gamma j(s,y)}{|x-y|}ds\,d\sigma_y=\vec{n}\wedge c.$$

Multiplying formally this equation by a vector test function $\varphi(t,x)$ on $\mathbb{R}_t^+\times\Gamma_x$ we obtain the variational formulation :

$$\int_0^\infty\{\iint_{\Gamma\times\Gamma}\frac{\varphi(t,x)}{|x-y|}\partial_t j(t-|x-y|,y)+\frac{div_\Gamma\varphi(t,x)}{|x-y|}[\int_0^{t-|x-y|}div_\Gamma j(s,y)ds]d\sigma_x d\sigma_y\}dt=$$
$$=4\pi\int_0^\infty\int_\Gamma\varphi(t,x)\,[\vec{n}\wedge c(t,x)]d\sigma_x\,dt$$

We show this bilinear form is continuous and coercive on suitable Hilbert spaces; its discrete approximation leads to a stable marching-in-time scheme.

2. Time-dependent problem. We begin this section with a brief description of the time functional framework according [1], [5]. We introduce :

$$H^{-1/2}(div,\Gamma)=\{c\in H^{-1/2}(\Gamma)^3, c.\vec{n}=0, div_\Gamma c\in H^{-1/2}(\Gamma)\}$$
$$H^{-1/2}(curl,\Gamma)=\{c\in H^{-1/2}(\Gamma)^3, c.\vec{n}=0, curl_\Gamma c\in H^{-1/2}(\Gamma)\}$$

with their natural norms $\|\cdot\|_{-1/2,div}$ and $\|\cdot\|_{-1/2,rot}$. Notice that if we identify $L^2(\Gamma)$ with its dual space, we can prove ([3], [8]) that $H^{-1/2}(div,\Gamma)$ is the dual space of $H^{-1/2}(curl,\Gamma)$ and conversely , $(H^{-1/2}(curl,\Gamma))'=H^{-1/2}(div,\Gamma)$.

Let E be an arbitrary Hilbert space and s and σ some reals, $\sigma>0$. Then $\mathscr{H}^s_\sigma(\mathbb{R}^+,E)$ is the set of E-valued distributions f with support in $\mathbb{R}^+$ satisfying :

$$e^{-\sigma t}\Lambda^s f\in L^2(\mathbb{R}_t;E),\ (\Lambda^s f)\hat{}(\omega)=(-i\omega)^s\hat{f}(\omega)$$

and $\hat{f}(\omega)$ is the Fourier-Laplace transform of f .We shall denote $\mathscr{H}^s_\sigma(\mathbb{R}^+,E)$ norm by $|.|_{\sigma,s,E}$:

$$|f|_{\sigma,s,E}=(\int_0^{+\infty}e^{-2\sigma t}\|\Lambda^s f\|_E^2dt)^{1/2}\equiv(\int_{-\infty+i\sigma}^{+\infty+i\sigma}|\omega|^{2s}\|\hat{f}(\omega)\|_E^2d\omega)^{1/2}.$$

For simplicity, we denote by $|f|_{\sigma,s,-1/2\,div}$ (resp. $|f|_{\sigma,s,-1/2\,curl}$) the norm of f in $\mathscr{H}^s_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$ (resp. $\mathscr{H}^s_\sigma(\mathbb{R}^+,H^{-1/2}(curl,\Gamma))$).

As we have already noted, relationships (1) and (3) lead to a time dependent integral equation relating j and c on the surface of the body; in fact this relation involves a pseudodifferential operator R on $\mathbb{R}_t\times\Gamma$ similar to Neuman operator for the wave equation; unfortunatly R belongs to the exotic class $OPS^1_{1/3,2/3}$ for which the properties of continuity on Sobolev spaces are unknown. According to Bamberger-HaDuong's idea [1], [5], we study the associated harmonic problem and deduce properties on R by using Fourier-Laplace transform. All the results in time domain follow from those in frequency domain. It suffices to apply an inverse Fourier-Laplace transform.

We prove that for c belonging to $\mathcal{H}^1_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$ the problem $(P_\pm)$ has a unique solution (E,H) which satisfies the following energy estimate :

$$\int_0^{+\infty} e^{-2\sigma t} \int_{\Omega_+\cup\Omega_-} (\,|E(t,x)|^2 + |H(t,x)|^2\,)\,dx\,dt \leq C_{\sigma_0,\Gamma}\,|c|^2_{\sigma,1,-1/2\,div} \quad \forall\sigma\geq\sigma_0>0$$

This electromagnetic field, represented by (1)-(2), is uniquely determined by density j . Then the integral equation for the transform $\hat{p}$ of $-\partial_t j$ translates into the integral equation for j :

$$Rj = \vec{n}\wedge c \qquad (4)$$

where R is defined by :

$$Rj = -\vec{n}\wedge(\vec{n}\wedge\partial_t Sj\,) - grad_\Gamma S\, div_\Gamma\Big(\int_0^t j(\tau)\,d\tau\Big)$$

and S is the retarded potential:

$$Sf(t,x) = \frac{1}{4\pi}\Big\{ \int_\Gamma \frac{f(t-|x-y|,y)}{|x-y|}\,d\gamma(y)\Big\},\; t\in\mathbb{R}^+, x\in\Gamma\,.$$

If datum c is in $\mathcal{H}^2_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$ for $\sigma>0$ then there exists a unique solution j in $\mathcal{H}^0_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$ of the integral equation. More generally if $c\in\mathcal{H}^{s+1}_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$ then $j\in\mathcal{H}^{s-1}_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$:

$$|j|_{\sigma,s-1,-1/2\,div} \leq C_{\sigma_0}\,|c|_{\sigma,s+1,-1/2\,div}\,,\; \forall\sigma\geq\sigma_0>0\,. \qquad (5)$$

At last we get a space-time variational formulation for the solution j of (8):

$$\forall p\in\mathcal{H}^1_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$$

$$\int_0^{+\infty} e^{-2\sigma t} \langle p(t,.), Rj(t,.)\rangle dt = \int_0^{+\infty} e^{-2\sigma t} \langle p(t,.), \vec{n}\wedge c(t,.)\rangle dt \qquad (6)$$

Hence the pseudodifferential operator R is linear continuous from $\mathcal{H}^2_\sigma(\mathbb{R}^+, H^{-1/2}(div,\Gamma))$ to $\mathcal{H}^0_\sigma(\mathbb{R}^+, H^{-1/2}(curl,\Gamma))$ and defines a bilinear form which satisfies the coercivity condition :

$$\int_0^{+\infty} e^{-2\sigma t} \langle p(t,.), Rp(t,.)\rangle dt \;\geq\; C\,|p|^2_{\sigma,-1,-1/2\,div}$$

related to the electromagnetic energy:

$$\int_0^{+\infty} e^{-2\sigma t} <j(t,.), Rj(t,.)> dt = \sigma \int_0^{+\infty} e^{-2\sigma t} (\|E(t)\|^2_{L^2(\Omega_+ \cup \Omega_-)} + \|H(t)\|^2_{L^2(\Omega_+ \cup \Omega_-)}) \, dt \, .$$

We close this section with a few words about the time regularity results. We did not obtain the best ones as we can see in (5). Because of the space-time decoupling in the employed technique, this method will probably not provide better results. However it will be more convenient for the numerical approach.

4. The discrete approximation of the variational problem. To calculate current j, induced on Γ, we give an approximation of variational problem (6) using a finite elements method in both time and space. We begin by discretizing in space.

We just present the formal elements of the space approximation.Therefore we shall discuss neither the substitution of the surface Γ by an approximate surface Γ_h nor the construction of the approximate space of $H^{-1/2}(div,\Gamma)$. For example, one can follow Nedelec's ideas [7] for the construction of Γ_h and the associated finite elements space is then described in [2].

Let V_h be a finite dimensional subspace of the space $H^{-1/2}(div,\Gamma)$. The unknown current is represented by an expansion of basis function φ_j^h of V_h as :

$$j(t,x) \approx j_h(t,x) = \sum_{j=1}^{N_h} \alpha_j(t)\, \varphi_j^h(x)$$

where $\alpha_j \in \mathscr{H}^1_\sigma(\mathbb{R}^+,\mathbb{R})$. The discrete problem consists in finding α_j such that :

$$\sum_{l=1}^{N_h} \int_0^{+\infty} e^{-2\sigma t} \beta_j(t) \iint_{\Gamma\times\Gamma} K_{jl}^{(1)}(x,y)\, \alpha'_l(t-|x-y|) + K_{jl}^{(2)}(x,y) \{ \int_0^{t-|x-y|} \alpha_l(\tau)\, d\tau \} d\gamma(x,y) dt$$

$$= -\int_0^{+\infty} e^{-2\sigma t} \beta_j(t) \int_\Gamma \varphi_j^h(x) (c_h(t,x) \wedge \vec{n}_x)\, d\gamma(x)\, dt \ , \ \forall j=1,\ldots,N_h$$

where β_j is a test function in $\mathscr{H}^1_\sigma(\mathbb{R}^+,\mathbb{R})$, c_h is an approximation of c in V_h and $K_{jl}^{(1)}$ and $K_{jl}^{(2)}$ are defined by :

$$K_{jl}^{(1)}(x,y) = \frac{\varphi_j^h(x)\varphi_l^h(y)}{4\pi\, |x-y|} \qquad K_{jl}^{(2)}(x,y) = \frac{div_\Gamma \varphi_j^h(x)\, div_\Gamma \varphi_l^h(y)}{4\pi\, |x-y|} \ .$$

In a second step, we choose a segmentation of the time positive axis into a regular grid $\{ t_n = n\ \Delta t\ ,\ n \in \mathbb{N} \}$. The functions of $\mathscr{H}^1{}_\sigma(\mathbb{R}^+,\mathbb{R})$ can be approximated by those of the subspace $\mathscr{H}^m(\Delta t, \mathbb{R})$ composed of polynomials of degree $m \geq 1$ in each time interval $I_n = (t_n, t_{n+1})$. The approximate current can therefore be written as :

$$j_{h\Delta t}(t,x)=\sum_{j=1}^{N_h} \alpha_{j\Delta t}(t)\,\varphi_j^h(x) \qquad (7)$$

where $\alpha_{j\Delta t}\in\mathcal{H}^m(\Delta t,\mathbb{R})$. We get the discrete approximation of problem (6)

$$\sum_{l=1}^{N_h}\int_0^{+\infty} e^{-2\sigma t}\beta_{j\Delta t}(t)\iint_{\Gamma\times\Gamma} K_{jl}^{(1)}(x,y)\,\alpha'_{l\Delta t}(t-|x-y|)+K_{jl}^{(2)}(x,y)\{\int_0^{t-|x-y|}\alpha_{l\Delta t}(\tau)d\tau\}d\gamma(x,y)dt$$

$$= -\int_0^{+\infty} e^{-2\sigma t}\,\beta_{j\Delta t}(t)\int_\Gamma \varphi_j^h(x)(c_h(t,x)\wedge\vec{n}_x)d\gamma(x)dt \ ,\ \forall j=1,\dots,N_h \qquad (8)$$

and all test functions $\beta_{j\Delta t}\in\mathcal{H}^m(\Delta t,\mathbb{R})$. In order to describe the method, let us consider the most simple choice of m which is $m = 1$. The $m>1$ case would use the same technical ideas as in calculations below.

Therefore $\alpha'_{l\Delta t}$ is a constant in I_n , denoted by a_l^n . If we choose the test functions as

$$\beta_{j\Delta t}(t)=t-t_{n-1} \quad for\ t\in I_{n-1}, =t_{n+1}-t \quad for\ t\in I_n, =0 \quad elsewhere\ ;$$

a simple substitution into (8) and some additional manipulations show that we can rewrite (8) in matrix form as :

$$M_0A^n=-\sum_{k=0}^{n-1} M_{n-k}A^k+B^n,\ 1\le n$$

where A^k is the vector of the unknowns $(a_1^k,\dots,a_{N_h}^k)^T$ and B^n is the vector corresponding to the right-hand side of (8). The matrix M_k is symmetric :

$$M_k^{jl}=\int_0^{\Delta t} e^{-2\sigma s}(\Delta t-s)\ \{\iint_{\Gamma\times\Gamma/s+t_{k-1}<|x-y|\le s+t_k}[K_{jl}^{(1)}(x,y)+\frac{1}{2}(s+t_k-|x-y|)^2K_{jl}^{(2)}(x,y)]\,d\gamma(x,y)$$

$$+\iint_{\Gamma\times\Gamma/|x-y|\le s+t_k}(s+t_k-|x-y|-\frac{\Delta t}{2})\,\Delta t\,K_{jl}^{(2)}(x,y)]\,d\gamma(x)d\gamma(y)\}\,ds$$

$$-e^{2\sigma\Delta t}\int_0^{\Delta t} e^{-2\sigma s}s\{\iint_{\Gamma\times\Gamma/s+t_{k-2}<|x-y|\le s+t_{k-1}}[K_{jl}^{(1)}(x,y)+\frac{1}{2}(s+t_{k-1}-|x-y|)^2K_{jl}^{(2)}(x,y)]\,d\gamma(x,y)$$

$$+\iint_{\Gamma\times\Gamma/|x-y|\le s+t_{k-1}}(s+t_{k-1}-|x-y|-\frac{\Delta t}{2})\,\Delta t\,K_{jl}^{(2)}(x,y)]\,d\gamma(x)d\gamma(y)\}\,ds$$

If the time sample interval Δt is small enough, then M_0 is positive definite and given $(B^n)_n$ computed from (8), this infinite system has a unique solution $(A^n)_n$ and this sheme is convergent :

THEOREM *Assume solution j of (4) satisfies*

$$j \in \mathcal{H}^{2}_{\sigma}(\mathbb{R}^{+}; H^{m_1}(div, \Gamma)) \cap \mathcal{H}^{m_2+1}_{\sigma}(\mathbb{R}^{+}; H^{-1/2}(div, \Gamma)),\ m_1 > 1,\ m_2 > 2;$$

then for any $\varepsilon \in]0,1/2]$ *, solution* $j_{h,\Delta t}$ *defined by (7) converge to* j *:*

$$\left| j - j_{h,\Delta t} \right|_{\sigma,-1,-1/2div} \leq C_{\varepsilon} \{ \left| c - c_{h,\Delta t} \right|_{\sigma,1,-1/2div} + \frac{h^{m_1-\varepsilon}}{\Delta t} \left| j \right|_{\sigma,2,m_1 div} + \Delta t^{m_2-2} \left| j \right|_{\sigma,m_2+1,-1/2div} \}.$$

Unfortunately this scheme is not marching in time because A^0 is not determined. Then, taking account the fact that M_1 is bigger than M_0 we make a *lumping* by putting:

$$\overline{M_1} = M_1 + e^{2\sigma\Delta t} M_0\ ,\ \overline{M_k} = M_k\ ,\ 2 \leq k\ ,\ B^n = \overline{B}^n,\ 1 \leq n\ ,$$

and we solve

$$\overline{M_1}\overline{A}^{0} = \overline{B}^{1},\quad \overline{M_1}\overline{A}^{n} = -\sum_{k=0}^{n-1} \overline{M}_{n+1-k}\overline{A}^{k} + \overline{B}^{n+1},\ 1 \leq n :$$

A single inversion of matrix $\overline{M_1}$ is required and this quasi-explicit marching-in-time scheme comes from a discrete variational problem: by putting

$$\overline{A}^{k} = (a^{k}_{1}, \ldots, a^{k}_{N_h})^{T},\ \ j_{h\Delta t}(t,x) = \sum_{j=1}^{N_h} \alpha_{j\Delta t}(t)\, \varphi^{h}_{j}(x) \qquad (9)$$

where

$$\alpha_{j\Delta t} \in \mathcal{H}^{1}_{\sigma}(\Delta t, \mathbb{R}),\ \alpha'_{l\Delta t} = a^{n}_{l}\ \ on\ I_n$$

$j_{h\Delta t}$ is the unique solution in $\mathcal{H}^{1}_{\sigma}(\Delta t, V_h) = \mathcal{H}^{1}_{\sigma}(\Delta t, \mathbb{R}) \otimes V_h$ satisfying

$$\forall\ \varphi \in \mathcal{H}^{1}_{\sigma}(\Delta t, V_h)\ ,$$

$$\int_0^{\infty} e^{-2\sigma t} \iint_{\Gamma\times\Gamma} \frac{\varphi(t,x)}{|x-y|} \partial_t j_{h\Delta t}(t-|x-y|,y) + \frac{div_{\Gamma}\varphi(t,x)}{|x-y|} \left(\int_0^{t-|x-y|} div_{\Gamma} j_{h\Delta t}(s,y)\,ds \right) d\gamma(x,y)dt$$

$$+ \int_0^{\infty} e^{-2\sigma t} \left(\left[\frac{t}{\Delta t}+1\right] \Delta t - t \right) \iint_{|x-y| \leq t - [\frac{t}{\Delta t}]\Delta t} \{ \frac{\partial_t \varphi(t,x)}{|x-y|} \partial_t j_{h\Delta t}(t-|x-y|, y) +$$

$$+ \frac{div_{\Gamma}\partial_t \varphi(t,x)}{|x-y|} \left(\int_{[\frac{t}{\Delta t}]\Delta t + |x-y|}^{t} \int_{[\frac{t}{\Delta t}]\Delta t}^{\tau - |x-y|} div_{\Gamma}\partial_t j_{h\Delta t}(s,y)\, ds\, d\tau \right) \} d\gamma(x,y)dt =$$

$$= 4\pi \int_0^{\infty} \int_{\Gamma} \varphi(t,x) \left[\vec{n} \wedge c_{h,\Delta t}(t,x) \right] d\gamma(x)dt$$

where $[x]$ notes the integer part of x : $[x] \in \mathbb{N},\ [x] \leq x < [x] + 1$.

The coercivity of this bilinear form on $\mathscr{H}^1_\sigma(\Delta t, V_h)$ leads to the scheme stability

THEOREM *Let $c_{h,\Delta t}$ be a consistant approximation of c in $\mathscr{H}^1_\sigma(\mathbb{R}^+;H^{-1/2}(div,\Gamma))$. . Then $j_{h,\Delta t}$ given by (9) satisfies:*

$$\left| j_{h,\Delta t} \right|_{\sigma,-1,-1/2div} \leq Cst\ , h \to 0\ ,\ \Delta t \to 0\ .$$

5. Numerical Experiments. We provide here numerical results for the two dimensional TM-scattering problem from an infinite perfectly conducting cylinder Ω_-: the electric field $E=(0,0,E_Z)$ is then parallel to the axis of the obstacle and satisfies:

$$(\partial_t^2 - \Delta_x) E = 0 \ \ in \ \ \mathbb{R}_t \times \Omega_+,$$

$$E_Z = E_Z^i \ on \ \mathbb{R}_t \times \Gamma\ ,\ \ E_Z(t,x) = \partial_t E_z(t,x) = 0 \ for \ \ t \leq 0\ ,\ x \in \Omega_+ .$$

Following an analogous procedure to that used for the 3D problem, one can represent the scattered field in terms of retarded potentials:

$$E_Z(t,x) = \frac{1}{2\pi} \int_\Gamma \int_0^{t-|x-y|} \frac{p(\tau,y)}{[(t-\tau)^2 - |x-y|^2]^{1/2}}\ d\tau\, d\Gamma_y\ , x \in \Omega_+$$

where p is related to the normal derivative of the total field

$$p = -\partial_n E_Z - \partial_n E_Z^i\ .$$

By writting the boundary condition on E_Z we obtain the integral equation

$$E_Z^i(t,x) = \frac{1}{2\pi} \int_\Gamma \int_0^{t-|x-y|} \frac{p(\tau,y)}{[(t-\tau)^2 - |x-y|^2]^{1/2}}\ d\tau\, d\Gamma_y\ , x \in \Gamma;$$

The unknown p is determined from the associated variational problem in $\mathscr{H}^1_\sigma(\mathbb{R}^+, H^{-1/2}(\Gamma))$:

$$\frac{1}{2\pi} \int_0^\infty e^{-2\sigma t} \iint_{\Gamma\times\Gamma} q(t,x) \int_0^{t-|x-y|} \frac{\partial_t p(\tau,y)}{[(t-\tau)^2 - |x-y|^2]^{1/2}}\ d\tau\, d\Gamma_y\, d\Gamma_x\, dt = \qquad (10)$$

$$= - \int_0^\infty \int_\Gamma q(t,x)\ \partial_t E_Z^i(t,x)\ d\Gamma_x\, dt.$$

Before beginning the approximation process, notice that in the variational formulation the time integral $\int_0^{t-|x-y|}$, following from the failure of Huygens' principle in the 2D case, leads us to similar characteristics as in the 3D Maxwell formulation.

We now turn to the discrete approximation of (10). As we have already noted, the chosen integral method uses the finite-element method in both time and space. First, the curve Γ is approximated by a union of straight segments

Γ_h.The finite dimensional subspace of $H^{-1/2}(\Gamma_h)$ is the space of functions defined over Γ_h whose restrictions to each segment is a constant. Next, the time positive axis is partitioned into intervals of lenght Δt. Since the unknown and the test function belong to $\mathscr{H}^1_\sigma(\mathbb{R}^+, H^{-1/2}(\Gamma))$, it seems that the approximation by continuous piecewise polynomial of degree one is natural. This approximation leads to a matrix system which is not a time stepping procedure. In order to overcome this numerical trouble, we do a lumping as in the 3D case. The new corresponding scheme is a marching in time algorithm that we call the " $\mathbb{P}_1$–$\mathbb{P}_1$ *scheme* ".

Using the fact that no derivative of the test function appears in (10) and that the time regularity results are not the best ones, one can try a zeroth-order time approximate test function, *ie.* a piecewise constant function in time.One obtains from this choice a marching in time scheme denoted by " $\mathbb{P}_0$–$\mathbb{P}_1$ *scheme*". We also remark that the time integral $\int_0^{t-|x-y|}$ allows us to take a weaker time regularity ($\mathbb{P}_0$ finite element) for the unknown. Then the approximate problem yields to an other marching in time procedure, the " $\mathbb{P}_0$–$\mathbb{P}_0$ *scheme* ".

Three representative geometries are considered: a circular cylinder, a square one and an elliptic one.The chosen incident fields are plane waves z-polarised and traveling in the negative x–direction.In order to assess the validity and the effiency of the three schemes, we use the principle of limiting amplitude : given an incident wave of frequency ω, $exp[i\omega(t-x)]$, as t tends to infinity, $exp(-i\omega t)\ E_Z(t,.)$ tends to the solution of time harmonic Maxwell's equations. In the following figures , we present the curves of the modulus of the current. The figures 1-2-3 show an excellent agreement of our results obtained by the " $\mathbb{P}_0$–$\mathbb{P}_0$ *scheme* " with those computed by CEA-CESTA 's boundary integral equation code.We test the long time stability of the schemes with an impulse excitation: in figure 4, we constat that the " $\mathbb{P}_0$–$\mathbb{P}_0$ *scheme* " provides the best accuracy without the oscillations that appear by using the " $\mathbb{P}_0$–$\mathbb{P}_1$ *scheme* ".

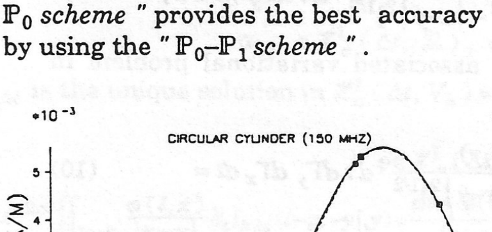

fig. 1
--- *time dependent code*
—— *harmonic code*

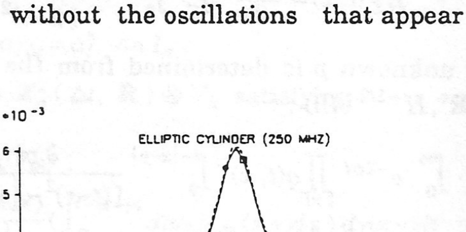

fig. 2
--- *time dependent code*
—— *harmonic code*

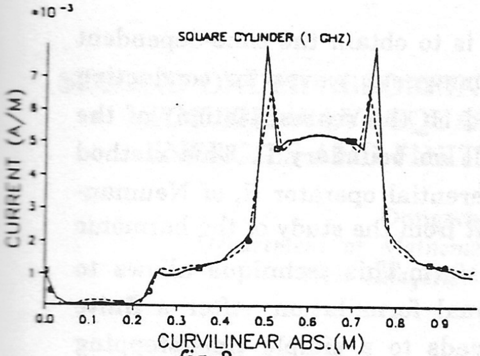

fig. 3

--- *time dependent code*

— *harmonic code*

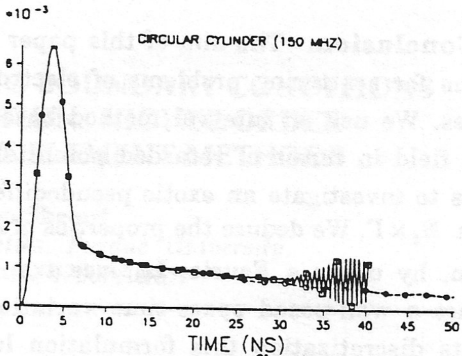

fig. 4

--- $\mathbb{P}_0$-$\mathbb{P}_0$ *scheme*

— $\mathbb{P}_0$-$\mathbb{P}_1$ *scheme*

The second method to evaluate the R.E.S. by a time dependent approach consists to take advantage of Majda's formula of representation of the scattering kernel [6] [8]; We compute by the $\mathbb{P}_0$-$\mathbb{P}_0$ scheme the scattered wave p, for incoming wave a regular breve impulse $\varphi(t-(x-x_0)/c)$ approximating the singular incoming wave $\delta(t-(x-x_0)/c)$ where δ is Dirac's distribution at 0:

$$\varphi(t)=\begin{cases} \frac{\omega}{2\pi}(1-\cos\omega t)\,,\ t\in[0,\frac{2\pi}{\omega}],\ \omega>>1,\\ 0 \ elsewehre. \end{cases}$$

By taking the Fourier transform $\hat{p}$ of p we get the backscattering amplitude by the formula

$$|s(k,u,-u)|^2=|\frac{i}{4}\left(\frac{2i}{\pi k}\right)^{1/2}\int_\Gamma e^{iku x}\,\hat{p}(kc,x)\,d\gamma(x)|^2$$

The main advantage of this method is the gain of computation time. The following table gives some R.E.S. in dbm^2 computed by the harmonic and time dependent codes.

frequence (MHZ)	HARMONIC CODE	$\mathbb{P}_0$–$\mathbb{P}_0$ *scheme*
100	5,3210	5,3277
150	5,1580	5,1569
250	5,0500	5,0253
300	5,0280	4,9537

6.Conclusion. The aim of this paper is to obtain the time-dependent solutions for scattering problems of electromagnetic waves for conducting obstacles. We use an integral method based on the representation of the electric field in terms of retarded potentials on boundary Γ. This method requires to investigate an exotic pseudodifferential operator R, of Neuman-type, on $\mathbb{R}_t \times \Gamma$. We deduce the properties of R from the study of the harmonic problem, by using a Fourier-Laplace transform.This technique allows to construct a well-posed space-time variational formulation. After a finite elements discretization this formulation leads to a simple time-stepping procedure and stability and convergence results are obtained. We present some numerical experimentations in 2D by computing the electric field scattered by a circular, elliptic, or square, infinite cylinder. Given a frequency ω we compare our time dependent code with an harmonic code of CEA/CESTA thanks to the principle of limiting amplitude: the results present an excellent agreement ; the computation of a short impulse shows the long time stability of this scheme; therefore we can expect a good efficiency of this method for the study of the scattering problems.

REFERENCES

[1] A. BAMBERGER, T.HA DUONG, *Math.Meth.Appl.Sci.*,8 (1986), pp.405-435 and 598-608 .

[2] A. BENDALI , *Thèse d'Etat*, Université Paris VI, 1984.

[3] M. CESSENAT, *Rapport Interne*, C.E.A./ C.E.L.V., 1984.

[4] R. DAUTRAY , J. L. LIONS , *Analyse mathématique et calcul numérique pour les sciences et les techniques* , Vol 1 and 5, Masson, 1985.

[5] T. HA - DUONG, *Thèse d'Etat*, Université Paris VI, France, 1987.

[6] A. MAJDA, *Comm. Pure Appl. Math.* 30 (1977), 165-194.

[7] J. C. NEDELEC , *Comp. Meth. Appl. Mech. Eng.* 8, (1976) , pp. 61-80.

[8] A. PUJOLS , *Thesis*, Bordeaux 1 University, 1991.