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Time dependent integral method for Maxwell's system with impedance boundary condition

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Abstract

We solve the problem of diffraction of an electromagnetic wave by a dissipative scatterer using a boundary integral method in time-domain directly. We prove the existence and uniqueness of the solution of this problem. We obtain the continuity and a relation of coercivity for the associated time-dependent formulation in this time functional framework. The discret approximation of the variational formulation leads to a stable marching-in-time scheme.

1 Introduction

The time dependent integral method was applied by T. HA DUONG [3] in 1987 for solving the waves equation in dimension 3D+1. He also defined a functional framework used by E. BECACHE [2] in elastic waves and by A. PUJOLS [6] for Maxwell's system in 2D+1. In 1993, I. TERRASSE [8] introduced new spaces coupling time and space and resolved numerically Maxwell equations for a perfect conductor. Here we study the case of a dissipative obstacle, with an impedance boundary condition, by considering a tridimensional object Ω^-, with regular bounded surface Γ and exterior normal $\vec{n}$, lighted by an incident wave (E^{inc}, H^{inc}) that hits the scatterer at $t = 0$. The scattered field (E, H) satisfies in $\Omega^+ = \mathbb{R}^3 \backslash \overline{\Omega^-}$ the Maxwell equations

$$\begin{cases} \overrightarrow{curl} E + \mu_o \partial_t H = \vec{0} & in\ \mathbb{R}^t \times \Omega^+ \\ \overrightarrow{curl} H - \varepsilon_o \partial_t E = \vec{0} & in\ \mathbb{R}^t \times \Omega^+ \\ div E = 0 = div H & in\ \mathbb{R}^t \times \Omega^+ \\ E(t,.) = 0 = H(t,.) & for\, t < 0 \end{cases}$$

We take on the surface Γ the Leontovitch condition with impedance Z :

$$\Pi_\Gamma E - Z\vec{n}\wedge H_{|\Gamma} = \vec{c} \qquad on\ \mathbb{R}^t\times\Gamma \tag{1}$$

The tangent vector $\vec{c}$ is given by the incident field :

$$\vec{c} = -\Pi_\Gamma E^{inc} + Z\vec{n}\wedge H^{inc}{}_{|\Gamma} \quad on\ \mathbb{R}^t\times\Gamma \ \ where\ \ \Pi_\Gamma f(x) = -\vec{n}(x)\wedge(\vec{n}(x)\wedge f_{|\Gamma})$$

The constants μ_o and ε_o are respectively the magnetic permeability and the electric permittivity. Now, we consider an interior problem defined by extending the field (E, H) into the interior domain Ω^-:

$$\begin{cases} \vec{curl}E + \mu_o\partial_t H = \vec{0} \ \ in\ \mathbb{R}^t\times\Omega^- \\ \vec{curl}H - \varepsilon_o\partial_t E = \vec{0} \ \ in\ \mathbb{R}^t\times\Omega^- \\ divE = 0 = divH \quad in\ \mathbb{R}^t\times\Omega^- \\ E(t,.) = 0 = H(t,.) \quad for\, t<0 \end{cases}$$

with the condition on the surface Γ :

$$\Pi_\Gamma E + Z\vec{n}\wedge H_{|\Gamma} = \vec{d} \quad on\ \mathbb{R}^t\times\Gamma \quad where\ \ \vec{d} = -\Pi_\Gamma E^{inc} - Z\vec{n}\wedge H^{inc}{}_{|\Gamma} \tag{2}$$

2 Representation by retarded potentials

The solution of $(P^\pm)$ can be represented in $\mathbb{R}^+\times(\Omega^+\cup\Omega^-)$ by :

$$E = -\mu_0 L\partial_t\vec{j} - \vec{grad}\,L\rho - \vec{curl}\,L\vec{m}, \ \ H = -\varepsilon_0 L\partial_t\vec{m} - \vec{grad}\,L\theta + \vec{curl}\,L\vec{j}$$

where L is the retarded potential of simple layer :

$$Lp(t,x) = \int_\Gamma \frac{p(t-|x-y|_c, y)}{4\pi|x-y|}\,d\Gamma(y)\ for\ t\in\mathbb{R}^+\ and\ x\in\mathbb{R}^3\backslash\Gamma$$

The term $|x-y|_c$ denotes the ratio $\frac{|x-y|}{c}$, and the surface currents $\vec{j}$ and $\vec{m}$ and charges ρ and θ are the jumps on $\mathbb{R}^+\times\Gamma$, connected by conservation of charge :

$$\vec{j} = \vec{n}\wedge H^+{}_{|\Gamma} - \vec{n}\wedge H^-{}_{|\Gamma}\ ,\ \ \vec{m} = \vec{n}\wedge E^-{}_{|\Gamma} - \vec{n}\wedge E^+{}_{|\Gamma}\ ,$$

$$\rho = \vec{n}.E^+{}_{|\Gamma} - \vec{n}.E^-{}_{|\Gamma}\ ,\ \ \theta = \vec{n}.H^+{}_{|\Gamma} - \vec{n}.H^-{}_{|\Gamma}$$

$$div_\Gamma\vec{j} + \varepsilon_0\partial_t\rho = 0\ ,\ \ div_\Gamma\vec{m} + \mu_0\partial_t\theta = 0\ \ on\ \mathbb{R}^+\times\Gamma \tag{3}$$

The representations become in $\mathbb{R}^+\times(\Omega^+\cup\Omega^-)$:

$$E(t,x) = -\mu_0 L\partial_t\vec{j} + \frac{1}{\varepsilon_0}\vec{grad}\,L\left(\int_0^t div_\Gamma\vec{j}(s,x)ds\right) - \vec{curl}\,L\vec{m} \tag{4}$$

$$H(t,x) = -\varepsilon_0 L\partial_t\vec{m} + \frac{1}{\mu_0}\vec{grad}\,L\left(\int_0^t div_\Gamma\vec{m}(s,x)ds\right) + \vec{curl}\,L\vec{j} \tag{5}$$

By adding and substracting boundary conditions (1) and (2), and by using (4) and (5), we obtain the integral equations on $\mathbb{R}^+\times\Gamma$:

$$2(R\vec{j} + \Pi_\Gamma Q\vec{m}) - Z\vec{j} = \vec{c} + \vec{d},\ \vec{n}\wedge\vec{m} - 2Z(\frac{\varepsilon_0}{\mu_0}\ \vec{n}\wedge R\vec{m} + \vec{n}\wedge Q\vec{j}) = \vec{c} - \vec{d} \qquad (6)$$

$$R\vec{f}(t,x) = -\mu_0 \Pi_\Gamma L\partial_t\vec{f}(t,x) + \frac{1}{\varepsilon}\ \vec{grad}_\Gamma L(\int_0^t div_\Gamma\vec{f}(s,x)ds)$$

$$Q\vec{f}(t,x) = \vec{n}_x \wedge \int_\Gamma \vec{grad}_x(\frac{1}{4\pi|x-y|}) \wedge \vec{f}(t-|x-y|_c, y)\, d\Gamma(y)$$

$$\vec{grad}_\Gamma f(x) = -\vec{n}(x)\wedge(\vec{n}(x)\wedge\vec{grad}_x f_{|\Gamma})$$

The resolution of the system (6) leads to determinate the field (E,H) in $\Omega^+\cup\Omega^-$ thanks to relations (4) and (5).

3 Functional framework

Before introducing the functional framework, we shall do some recalls about the Fourier-Laplace transform. Let E be an Hilbert space, we note $\mathscr{D}_+'(E)$ the set of E-valued distributions, and $\mathscr{S}'(E)$ the set of E-valued tempered distributions with support in $\mathbb{R}^+$. For real σ, $\sigma>0$, we can define:

$$LT(\sigma,E) = \{\, T\in\mathscr{D}_+'(E),\ e^{-\sigma t}T\in\mathscr{S}_+'(E)\}$$

and the Fourier-Laplace transform $\hat{T}$ of T:

$$\hat{T}(\omega) = \mathscr{F}(e^{-\sigma t}T)(\eta) = \int_{-\infty}^{+\infty} e^{i\omega t}\,T(t)\,dt \quad T\in L^1(\mathbb{R})$$

where $\mathscr{F}$ is the usual Fourier transform and the frequency $\omega = \eta + i\sigma$.
For $r\in\mathbb{R}$, we define the spaces for $s\in\mathbb{R}$ and $r\in\mathbb{R}$:

$$H^r(div,\Gamma) = \{\, f\in H^r(\Gamma) : f.\vec{n} = 0,\ div_\Gamma f\in H^r(\Gamma)\,\}$$

$$H^r(\vec{curl},\Gamma) = \{\, f\in H^r(\Gamma) : f.\vec{n} = 0,\ \vec{curl}_\Gamma f\in H^r(\Gamma)\,\}$$

$$H^s_\sigma(\mathbb{R}^+,H^r(div,\Gamma)) = \{\, f\in LT(\sigma,H^r(div,\Gamma)),\ \int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s}\ \|\hat{f}(\omega,.)\|^2_{r,\omega,div_\Gamma}\,d\omega < +\infty\ \},$$

$$\|f\|^2_{s,\sigma,H^r(div,\Gamma)} = \frac{1}{2\pi}\int_{-\infty+i\sigma}^{+\infty+i\sigma} |\omega|^{2s}\ \|\hat{f}(\omega,.)\|^2_{r,\omega,div_\Gamma}\,d\omega$$

where $\quad \forall\,\hat{f}\in H^r(div,\Gamma),\quad \|\hat{f}(\omega,.)\|^2_{r,\omega,div_\Gamma} = \|\hat{f}^{\,\omega}(\omega,.)\|^2_{H^r(div,|\omega|\Gamma)}$

and $\quad \forall\, y\in|\omega|\Gamma,\quad \hat{f}^{\,\omega}(\omega,y) = \frac{1}{|\omega|}\hat{f}(\omega,\frac{y}{\omega})$

These norms are equivalent to the usual norms and we have the

Proposition 1. *For all* $|\omega| \geq \sigma$, *and* $r \geq 0$ *we have :*

$$C(\sigma)^{-(r+1)} \| \hat{f} \|_{r,\omega,div_\Gamma} \leq \| \hat{f} \|_{H^r(div,\Gamma)} \leq C(\sigma)^{(r+1)} |\omega|^{(r+1)} \| \hat{f} \|_{r,\omega,div_\Gamma}$$

$$C(\sigma)^{-(r+1)} |\omega|^{-r} \| \hat{f} \|_{-r,\omega,div_\Gamma} \leq \| \hat{f} \|_{H^{-r}(div,\Gamma)} \leq C(\sigma)^{(r+1)} |\omega| \, \| \hat{f} \|_{-r,\omega,div_\Gamma}$$

where $C(\sigma) = sup(\frac{1}{\sigma}, 1)$

We have the same result for $H^r(\vec{curl},\Gamma)$ by replacing in Proposition 1, *div* by $\vec{curl}$.

4 Variational problem

Adopting HA-DUONG's approach [3], we study the associated harmonic problem to deduce properties of R and Q by using Fourier-Laplace transform.Therefore, the system of integral equations (6) becomes :

$$\begin{cases} 2\,(R_\omega \hat{j} - 2\,\Pi_\Gamma Q_\omega \hat{m}) - Z\hat{j} = (\hat{c} + \hat{d}) \\ \vec{n} \wedge \hat{m} - 2\,Z\,(\frac{\varepsilon_0}{\mu_0}\, \vec{n} \wedge R_\omega \hat{m} + \vec{n} \wedge Q_\omega \hat{j}) = (\hat{c} - \hat{d}) \end{cases} \tag{7}$$

where $\hat{j}$ and $\hat{m}$ are the jumps of $\hat{H} \wedge \vec{n}$ and $-\hat{E} \wedge \vec{n}$ through Γ respectively, $\hat{c} = -\Pi_\Gamma \hat{E}^{inc} + Z\vec{n} \wedge \hat{H}^{inc}|_\Gamma$ and $\hat{d} = -\Pi_\Gamma \hat{E}^{inc} - Z\vec{n} \wedge \hat{H}^{inc}|_\Gamma$ R_ω and Q_ω are the operators

$$\begin{cases} R_\omega \hat{f}(x) = \mu_0 \int_\Gamma \frac{e^{i\omega|x-y|}}{4\pi|x-y|} \hat{f}(y)\, d\Gamma(y) - \frac{1}{\varepsilon_0}\, \vec{grad}_\Gamma \int_\Gamma \frac{e^{i\omega|x-y|}}{4\pi|x-y|}\, div_\Gamma \hat{f}(y)\, d\Gamma(y) \\ Q_\omega \hat{f}(x) = \int_\Gamma \vec{grad}_x (\frac{e^{i\omega|x-y|}}{4\pi|x-y|}) \wedge \hat{f}(y)\, d\Gamma(y) \end{cases}$$

We obtain the variational problem :

$$a_\omega((\hat{j},\hat{m}),(\hat{j}',\hat{m}')) = \frac{1}{2} < (\hat{c}+\hat{d}), \hat{j}' > - \frac{1}{2Z} < (\hat{c}-\hat{d}), \vec{n} \wedge \hat{m}' > \tag{8}$$

$$a_\omega((\hat{j},\hat{m}),(\hat{j}',\hat{m}')) = \frac{1}{2Z} \int_\Gamma \hat{m}(x) . \overline{\hat{m}'(x)}\, d\Gamma(x) + \frac{1}{2Z} \int_\Gamma \hat{j}(x) . \overline{\hat{j}'(x)}\, d\Gamma(x) - < R_\omega \hat{j}, \hat{j}' >$$

$$- < \frac{\varepsilon_0}{\mu_0} R_\omega \hat{m}, \hat{m}' > + < \Pi_\Gamma Q_\omega \hat{m}, \hat{j}' > - < \vec{n} \wedge Q_\omega \hat{j}, \vec{n} \wedge \hat{m}' > \tag{9}$$

Proposition 2. *For* $\mathcal{I}m(\omega) = \sigma \geq \sigma_0 > 0$, R_ω ,Q_ω *satisfy :*

$$\| R_\omega \hat{f} \|_{-1/2,\omega,\vec{curl}_\Gamma} \leq C(\Gamma,\sigma_0) \| \hat{f} \|_{-1/2,\omega,div_\Gamma} \qquad \forall \hat{f} \in H^{-1/2}(div,\Gamma)$$

$$\| (\vec{n} \wedge Q_\omega + \frac{I}{2}) \hat{f} \|_{-1/2,\omega,div_\Gamma} \leq C(\Gamma,\sigma_0) \| \hat{f} \|_{-1/2,\omega,div_\Gamma} \qquad \forall \hat{f} \in H^{-1/2}(div,\Gamma)$$

Proposition 3. *The associated sesquilinear form* a_ω *satisfies the coercivity condition* $\forall\, (\hat{j},\hat{m}) \in (L^2(\Gamma) \cap H^{-1/2}(div,\Gamma))^2$:

$$\mathcal{R}e\; a_\omega(\hat{j},\hat{m}) \geq C\, [\, \|\hat{j}\|^2_{0,\omega,\Gamma} + \| \hat{m} \|^2_{0,\omega,\Gamma} + \frac{\sigma_0}{|\omega|} (\|\hat{j}\|^2_{-1/2,\omega,div_\Gamma} + \| \hat{m} \|^2_{-1/2,\omega,div_\Gamma})\,] \tag{10}$$

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Hence, the harmonic problem (7) can be solved by the standard variational method:

Theorem 4. *Let $\mathcal{R}e(Z)>0$ and $\mathcal{I}m(\omega)>0$, $\hat{c}-\hat{d}$ and $\hat{c}+\hat{d}$ be in $L^2(\Gamma)\cup H^{-1/2}(div,\Gamma)$ and $L^2(\Gamma)\cup H^{-1/2}(c\vec{u}rl,\Gamma)$ respectively. Then problem (7) has a unique solution $(\hat{j},\hat{m}) \in (L^2(\Gamma)\cap H^{-1/2}(div,\Gamma))^2$ and we have :*

$$\{ \|\hat{j}\|^2_{0,\omega,\Gamma} + \|\hat{m}\|^2_{0,\omega,\Gamma} + C(\Gamma)\frac{\sigma_0}{|\omega|}(\|\hat{j}\|^2_{-1/2,\omega,div_\Gamma} + \|\hat{m}\|^2_{-1/2,\omega,div_\Gamma})\} \leq$$

$$\leq C_1(\mathcal{R}e(Z))\,(\|\hat{c}+\hat{d}\|^2_{0,\omega,\Gamma} + \|\hat{c}-\hat{d}\|^2_{0,\omega,\Gamma} + \|\hat{c}+\hat{d}\|^2_{-1/2,\omega,curl_\Gamma} + \|\hat{c}-\hat{d}\|^2_{-1/2,\omega,div_\Gamma})$$

Applying the inverse Fourier-Laplace transform to the solution of harmonic problem, we can deduce :

Theorem 5. *For $\mathcal{R}e(Z)>0, \vec{c}-\vec{d}$, $\vec{c}+\vec{d}$ be in $H^{s_1}_\sigma(\mathbb{R}^+,L^2(\Gamma))\cup H^{s_2+1/2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$ and $H^{s_1}_\sigma(\mathbb{R}^+,L^2(\Gamma))\cup H^{s_2+1/2}_\sigma(\mathbb{R}^+,H^{-1/2}(c\vec{u}rl,\Gamma))$ respectively, s_1, $s_2 \in \mathbb{R}$, Problem (6) has a unique solution $(\vec{j},\vec{m}) \in (H^{s_1}_\sigma(\mathbb{R}^+,L^2(\Gamma))\cap H^{s_2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma)))^2$.*

Parseval formula applied to (8) leads to the space-time variational formulation of time dependent problem for $(\vec{j}\,', \vec{m}') \in (H^{-s_1}_\sigma(R^+,L^2(\Gamma))\cap H^{1-s_2}_\sigma(R^+,H^{-1/2}(div,\Gamma)))^2$

$$a((\vec{j},\vec{m}),(\vec{j}\,',\vec{m}')) = \frac{1}{2}\int_0^\infty e^{-2\sigma t}(<\frac{1}{Z}(\vec{c}-\vec{d}),\vec{n}\wedge\vec{m}'> - <\vec{c}+\vec{d},\vec{j}\,'>)\,dt \tag{11}$$

$$a((\vec{j},\vec{m}),(\vec{j}\,',\vec{m}')) = \int_0^\infty e^{-2\sigma t}\{\int_\Gamma \frac{1}{2Z}\vec{m}(t,x).\overline{\vec{m}'(t,x)} + \frac{Z}{2}\vec{j}(t,x).\overline{\vec{j}\,'(t,x)}\,d\Gamma(x)$$

$$-<R\vec{j},\vec{j}\,'> - <\frac{\varepsilon_0}{\mu_0}R\vec{m},\vec{m}'> + <\Pi_\Gamma Q\vec{m},\vec{j}\,'> + <\vec{n}\wedge Q\vec{j},\vec{n}\wedge\vec{m}'>\}\,dt$$

The braket denotes the duality $H^s_\sigma(\mathbb{R}^+,H^{-1/2}(c\vec{u}rl,\Gamma))\times H^{-s}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma))$. The continuity of R_ω and Q_ω and coercivity relation (10) imply the continuity of a on $(H^{s_1}_\sigma(\mathbb{R}^+,L^2(\Gamma))\cap H^{s_2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma)))^2 \times (H^{-s_1}_\sigma(\mathbb{R}^+,L^2(\Gamma))\cap H^{1-s_2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma)))^2$ and the coercivity relation for $(\vec{j},\vec{m}) \in (H^0_\sigma(\mathbb{R}^+,L^2(\Gamma))\cap H^{1/2}_\sigma(\mathbb{R}^+,H^{-1/2}(div,\Gamma)))^2$:

$$a((\vec{j},\vec{m}),(\vec{j},\vec{m})) \geq$$

$$\geq C\,(\|\vec{j}\|^2_{0,\sigma,L^2(\Gamma)} + \|\vec{m}\|^2_{0,\sigma,L^2(\Gamma)} + \|\vec{m}\|^2_{-1/2,\sigma,H^{-1/2}(div,\Gamma)} + \|\vec{j}\|^2_{-1/2,\sigma,H^{-1/2}(div,\Gamma)}) \tag{12}$$

5 Approximation of variational problem (13)

We first make a space approximation.We construct an approximate surface Γ_h of Γ composed by regular triangles. Hence, we consider the edge element family of RAVIART-THOMAS [7] divergence conforming space V_h consisting of polynomials of degree one. P^{-1} denotes the inverse map of $(P_{|\Gamma_h}: \Gamma_h \to \Gamma)$. We can also define the space : $\tilde{V}_h = \{ \tilde{\varphi} = \varphi \circ P^{-1};\ \varphi \in V_h\}$. Then, the unknowns $\vec{j}$ and $\vec{m}$ are represented by an expansion of basis function $\vec{\varphi_j}$ for $j=1,Nar$ of $\tilde{V}_h$, where Nar is the total number of ridges, as:

$$\vec{j}(t,y) \approx \vec{j}_h(t,y) = \sum_{j=1}^{Nar} \alpha_j(t)\vec{\varphi_j}(y) \quad \vec{m}(t,y) \approx \vec{m}_h(t,y) = \sum_{k=1}^{Nar} \beta_k(t)\vec{\varphi_k}(y)$$

where α_j and $\beta_k \in H_\sigma^{r_1}(\mathbb{R}^+,\mathbb{R})$, $r_1 \in \mathbb{R}$.

We choose the test functions as :

$$\vec{j}'(t,y) \approx \vec{j}'_h(t,y) = \sum_{i=1}^{Nar} \eta_i(t)\vec{\varphi_i}(y) \qquad \vec{m}'(t,y) \approx \vec{m}'_h(t,y) = \sum_{l=1}^{Nar} \gamma_l(t)\vec{\varphi_l}(y)$$

where η_i and $\gamma_l \in H_\sigma^{r_2}(\mathbb{R}^+,\mathbb{R})$, $r_2 \in \mathbb{R}$.

In a second step, the positive time axis is divided into subintervals $I_k = [t_k,t_{k+1}[$ of length Δt. The function of $H_\sigma^r(\mathbb{R}^+,\mathbb{R})$ is approximated by those of the subspace $H_\sigma^m(\Delta t, \mathbb{R})$, $m \in \mathbb{N}$, of polynomials of degree $m \geq r$ in each time interval I_k :

$$H_\sigma^m(\Delta t,\mathbb{R}) \;=\; \{f \in H_\sigma^r(\mathbb{R}^+,\mathbb{R})\ ;\ f_{[t_k,t_{k+1}[} \in \mathbb{P}^m\}$$

The functions α_j, β_k, η_i and γ_l are approximated by :

$$\alpha_j(t) \approx \sum_{m\geq 1} \chi^m(t)X_j^m \qquad \beta_k(t) \approx \sum_{m\geq 1} \chi^m(t)Y_k^m \quad for\ j,k = 1,Nar$$

$$\eta_i(t) \approx \chi^n(t) \qquad \gamma_l(t) \approx \chi^n(t) \quad for\ i,l = 1,Nar, \quad \chi^m(t) = \begin{cases} 1 & if\ t \in [t_{m-1},t_m[\\ 0 & elsewhere \end{cases}$$

Now we approache the surface Γ. We take also $\sigma = 0$ and we put $p=n-m$. We obtain a symetrical matricial system :

$$\begin{cases} M^0\vec{U}^1 = \vec{SM}^1 \\ M^0\vec{U}^l = -\sum_{p=1}^{l-1} M^{l-p}\,\vec{U}^p + \vec{SM}^l \ for\ l \geq 2 \end{cases}$$

$$with \qquad U^p = \begin{bmatrix} \vec{X}^p \\ \vec{Y}^p \end{bmatrix} \qquad\qquad M^p = \begin{bmatrix} A1^p + C^p & A2^p \\ A2^p & A3^p + D^p \end{bmatrix}$$

$$A1_{ij}^p = -\int_{\Gamma_h}\int_{\Gamma_h} \mu_0 K_{ij}^{(1)}(x,y)[\int_{t_{n-1}}^{t_n} \chi^{m\prime}(t-|x-y|_c) + \frac{1}{\varepsilon_0} K_{ij}^{(2)}(x,y)(\int_0^{t-|x-y|_c} \chi^m(r)dr)dt]d\Gamma(x)d\Gamma(y)$$

$$A2_{ij}^p = -\int_{\Gamma_h}\int_{\Gamma_h} K_{ij}^{(3)}(x,y) \int_{t_{n-1}}^{t_n} \chi^m(t-|x-y|_c)dt\, d\Gamma(x)d\Gamma(y)$$

$$A3^p_{ij} = \int_{\Gamma_h}\int_{\Gamma_h} \varepsilon_0 K^{(1)}_{ij}(x,y)[\int_{t_{n-1}}^{t_n} \chi^{m\prime}(t-|x-y|_c)+\frac{1}{\mu_0} K^{(2)}_{ij}(x,y)(\int_0^{t-|x-y|_c} \chi^m(r)dr)dt]d\Gamma(x)\,d\Gamma(y)$$

$$C^p_{ij} = -\frac{Z}{2}\int_{t_{n-1}}^{t_n} \chi^m(t)\,dt \int_{\Gamma_h} \vec{\varphi_i}(x)\,.\,\vec{\varphi_j}(x)\,d\Gamma(x)\ ,\ D^p_{ij} = \frac{1}{2Z}\int_{t_{n-1}}^{t_n} \chi^m(t)\,dt \int_{\Gamma_h} \vec{\varphi_i}(x)\,.\,\vec{\varphi_j}(x)\,d\Gamma(x)$$

$$K^{(1)}_{ij}(x,y) = \frac{\vec{\varphi_i}(x)\,.\,\vec{\varphi_j}(y)}{4\pi|x-y|}\ ,\ K^{(2)}_{ij}(x,y) = \frac{div_\Gamma\vec{\varphi_i}(x)\,.\,div_\Gamma\vec{\varphi_j}(y)}{4\pi|x-y|}$$

$$K^{(3)}_{ij}(x,y) = \overrightarrow{grad}_x\frac{1}{4\pi|x-y|}\,.\,\vec{\varphi_i}(x)\wedge\vec{\varphi_j}(y)$$

The discret problem is a quasi-explicit marching-in-time scheme : a single inversion of the matrix M^0 is required.

6 Numerical results

We have tested this scheme on a sphere of radius 0.25m approached by 80 triangles. We represent the solution computed for different values of $CFL = c\frac{\Delta t}{\Delta x}$ at the lighted point (solid line) and at the hidden point (dotted line) of the object. The figures present the electric and magnetic currents for $CFL=0.5$, the frequency $F=200\ MHz$ and the impedance $Z=1$ for a sinusoidal wave (at the top) and for an impulsion (at the bottom). We take always 10 grid points/wavelengh. Increasing the number of triangles, frequency can be taken higher. Results have been valued using three tests :

- the theorem of limited amplitude,
- the Weston's theorem for a scatterer with a symetry of revolution,
- another program wich calculates the currents for a scatterer with 2D axial symetry.

7 Conclusion

We have solved Maxwell's system for dissipative obstacles by an integral method based of the representation of the electromagnetic field by retarded potentials on the surface Γ. By using Fourier-Laplace transform, we obtain a well-posed variational problem, the continuity and a relation of coercivity of the associated sesquilinear form. We approach another sesquilinear form and we obtain a numerical stability. The scheme is directly solved.Therefore, we can conclude that our scheme is perfectly robust and stable.

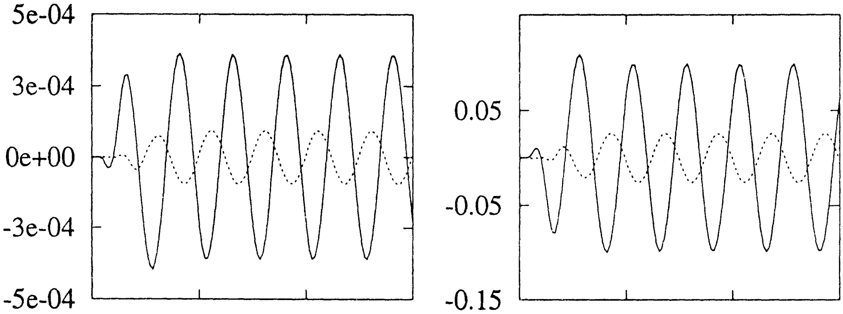

Figure 1 : Electric and magnetic currents for frequency 200 *MHz* and $Z = 1$.

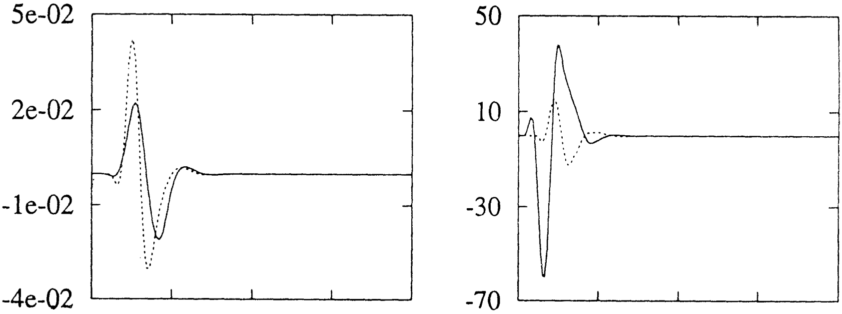

Figure 2 : Electric and magnetic currents for short impulse and $Z = 1$.

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