Numerical Analysis/Partial Differential Equations

Coupling of a multilevel fast multipole method and a microlocal discretization for the 3-D integral equations of electromagnetism ☆

Couplage d'une méthode multipôle rapide et d'une méthode de discrétisation microlocale pour les équations intégrales de l'électromagnétisme en 3-D

Alain Bachelot [a], Eric Darrigrand [b], Katherine Mer-Nkonga [c]

[a] *Université Bordeaux 1, MAB, 351, cours de la Libération, 33405 Talence cedex, France*
[b] *University of Delaware, Dept. of Mathematical Sciences, Newark, DE 19716, USA*
[c] *CEA/CESTA, BP 2, 33114 Le Barp, France*

Abstract

The aim of this work is to propose an accurate and efficient numerical approximation for high frequency diffraction of electromagnetic waves. In the context of the boundary integral equations presented in F. Collino and B. Després, to be published in J. Comput. Appl. Math., the strategy we propose combines the microlocal discretization (T. Abboud et al., in: Third International Conference on Mathematical Aspects of Wave Propagation Phenomena, SIAM, 1995, pp. 178–187) and the multilevel fast multipole method (J.M. Song, W.C. Chew, Microw. Opt. Tech. Lett. 10 (1) (1995) 14–19). This leads to a numerical method with a reduced complexity, of order $O(N^{4/3} \ln(N) + N_{iter} N^{2/3})$, instead of the complexity $O(N_{iter} N^2)$ for a classical numerical iterative solution of integral equations. Computations on an academic geometry show that the new method improves the efficiency, for a solution with a good level of accuracy. ***To cite this article: A. Bachelot et al., C. R. Acad. Sci. Paris, Ser. I 336 (2003).***
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Résumé

Nous proposons une résolution précise et rapide du problème de diffraction d'onde électromagnétique à haute fréquence. Dans le cadre de la modélisation du problème à l'aide des équations intégrales proposées par B. Després et reformulées dans l'article de F. Collino et B. Després, qui sera publié dans J. Comput. Appl. Math., la stratégie proposée combine la discrétisation microlocale de Abboud, Nédélec et Zhou (dans : Third International Conference on Mathematical Aspects of Wave Propagation Phenomena, SIAM, 1995, pp. 178–187) et la méthode multipôle rapide multi-niveaux (J.M. Song, W.C. Chew,

Microw. Opt. Tech. Lett. 10 (1) (1995) 14–19). On obtient alors une méthode numérique ayant une complexité d'ordre $O(N^{4/3} \ln(N) + N_{\text{iter}} N^{2/3})$ au lieu de la complexité en $O(N_{\text{iter}} N^2)$ pour une résolution itérative numérique classique des équations intégrales. Les résultats numériques sur une géométrie académique prouvent l'efficacité de la nouvelle méthode, pour une solution ayant un bon niveau de précision. ***Pour citer cet article : A. Bachelot et al., C. R. Acad. Sci. Paris, Ser. I 336 (2003).***

Version française abrégée

On considère le problème de la diffraction d'ondes électromagnétiques dans le contexte des hautes fréquences. Le problème est modélisé par les équations de Maxwell dans le domaine entourant l'objet tridimensionnel et par une condition d'impédance de type Léontovitch posée sur la surface Γ de l'objet. L'objet diffractant est supposé convexe et éclairé par une onde plane. Le problème est ramené à une équation intégrale sur Γ, ce qui permet de réduire la taille du système linéaire à résoudre. Cependant, celui-ci est dense, mal conditionné et de taille proportionnelle à $k^2 \times k^2$ où k est le nombre d'onde, ce qui implique une résolution numérique ayant un coût prohibitoire pour les problèmes à hautes fréquences.

Pour apporter des solutions à ces problèmes de coût, on propose de combiner trois méthodes ou formulations modernes ayant chacune des propriétés attrayantes : d'une part une modélisation à l'aide des équations intégrales proposées par Després [3], qui aboutit à un système linéaire (1) ayant de bonnes propriétés pour la résolution itérative ; d'autre part une méthode de discrétisation microlocale proposée dans [1,9] pour des objet convexes, permettant de réduire la taille du système à une taille de l'ordre de $k^{2/3} \times k^{2/3}$ par une estimation de la phase de l'inconnue (2) ; enfin, la méthode multipôle rapide multi-niveaux [7] qui permet d'accélérer le calcul de la matrice du système (3), dont le coût initial est d'ordre $O(N^2)$.

Le couplage de méthodes proposé aboutit à une utilisation originale de la méthode multipôles et à un nouvel algorithme multi-niveaux (5)–(7). On obtient une méthode ayant une complexité en $O(N^{4/3} \ln(N) + N_{\text{iter}} N^{2/3})$. Notons λ la longueur d'onde. Des résultats numériques sur la diffraction d'une sphère de taille 26λ en condition de conducteur parfait, et de taille 38λ en condition d'impédance $Z = 1$ (impédance du vide), ont été obtenus en discrétisant l'inconnue sur un maillage grossier dont la taille moyenne des arêtes est 1.6λ au lieu du classique $\lambda/10$. Cette méthode a été développée précédemment dans le contexte de l'équation de Helmholtz avec l'utilisation d'une méthode multipôle à un niveau dans [4].

1. Introduction

Let us consider the diffraction of an electromagnetic wave by a homogeneous 3-D bounded obstacle. In this paper, we focus on the problem of diffraction of high frequency incident waves, which has applications, for example, in Radar Cross Section studies. Classical numerical modelizations of such high frequency problems leads to expensive computer calculations and our purpose is to propose an efficient numerical modelization and solution.

The problem is modeled by a boundary integral equation. Using appropriate finite element methods, this formulation leads to an accurate numerical scheme for the exterior problem. After discretization, we have to solve a dense linear system with a size N proportional to the wave number $k = 2\pi f/c$ where f is the frequency of the incident wave and c the speed of light; the complexity of the dense matrix inversion is $O(k^4)$. This makes the method unusable for high frequency problems. The use of an *a priori* knowledge of the behavior of the phase of the exact solution allows to reduce the size of the system; for example, the microlocal discretization due to Abboud et al. [1] and Zhou [9] which can be used in the case of convex obstacles and plane incident waves, reduces the size of the system from $O(k^2 \times k^2)$ to $O(k^{2/3} \times k^{2/3})$. However, the cost of the matrix calculation is heavy, of order $O(N^2)$, and the stationary phase theory is used to sparsify it, indeed in the 2-D case. In the 3-D case, difficulties not yet solved, to our knowledge, appear when the stationary phase theory is used.

Another method, for high frequencies or problems with large obstacles, is the multilevel fast multipole method (MLFMM) [7]. In a classical context, it is used to reduce the cost of matrix-vector products of iterative methods

and has a complexity of order $O(N_{\text{iter}} N \ln(N))$. We propose to combine the microlocal discretization with the MLFMM, the latest method being used to accelerate the matrix calculation, which is an original use of the MLFMM. We choose to develop this coupling of methods in the context of the boundary integral equations proposed by Després [3]. Indeed, the discretization of these equations leads to a linear system with good properties for iterative solutions. A classical use of the MLFMM for this integral formulation has been proposed in [6]. The method proposed here for Maxwell's equations has been developed in the context of Helmholtz's equation using a one-level multipole method in [4].

The paper is organized as follows: in Section 2, we present briefly the integral equations proposed by Després and its numerical discretization and iterative solution. In Section 3, we present the microlocal discretization of Abboud et al. and pay attention to a particularity of this method which is the use of two imbricated meshes of the surface of the obstacle. In Section 4, we describe our coupling of the microlocal discretization and the MLFMM, and the new complexity. Section 5 is devoted to numerical results.

2. Després's integral equations

The electromagnetic scattering problem is modeled by Maxwell's equations with a Léontovitch type Impedance Boundary Condition (IBC). Let Ω be a bounded domain with boundary Γ, and let $\mathbf{n}$ denote the unit vector to Γ directed into the exterior of Ω. The IBC is characterized by its reflexion coefficient $R = (\text{Id} - Z)(\text{Id} + Z)^{-1}$ where Z is the relative impedance of the boundary. The Després's integral equations can be derived from two ways, one of them based on combinations of the classical integral representations and the IBC, and the other one on the minimization of a quadratic functional with linear constraints [3]. Let $\mathbf{J} = \mathbf{n} \wedge \mathbf{H}_{|\Gamma}$ and $\mathbf{M} = -\mathbf{n} \wedge \mathbf{E}_{|\Gamma}$ be the equivalent currents, where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic scattered fields in Heaviside–Lorentz units; the integral equations can be written

$$\begin{pmatrix} (1+\beta)I_d + (\mathcal{A}^\infty)^* \mathcal{A}^\infty & -\mathcal{T}^* + i\beta I_d \\ \mathcal{T} - i\beta I_d & \beta I_d + (\mathcal{A}^\infty)^* \mathcal{A}^\infty \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix} + \mathcal{N}_R \mathbf{U} = \begin{pmatrix} \mathbf{G} \\ \mathbf{0} \end{pmatrix}, \tag{1}$$

where $\mathbf{U} = (u_1, u_2)^{\text{T}} = (\sqrt{i}\mathbf{J}, \sqrt{i}^{-1}\mathbf{M})^{\text{T}}$ is called the primal unknowns and $\mathbf{V} = (\mathbf{J}', \mathbf{M}')^{\text{T}}$ the dual unknowns, due to the underlying minimization problem. In (1), $\beta \in]0, 1[$ and the integral operators are defined by

$$\mathcal{T} = \begin{pmatrix} T_r & K_r - \frac{1}{2}\mathbf{n}\wedge \\ K_r - \frac{1}{2}\mathbf{n}\wedge & T_r \end{pmatrix}, \qquad (\mathcal{A}^\infty)^* \mathcal{A}^\infty = \begin{pmatrix} T_i & K_i \\ K_i & T_i \end{pmatrix},$$

where T_r, K_r (resp. T_i, K_i) are the real part (resp. imaginary part) of the classical integral operators of the Stratton–Chu representation formulae. $\mathcal{N}_R$ is a simple integral operator which satisfies $\mathcal{N}_R \equiv 0$ if $R = 0$, which corresponds to the perfectly absorbing case. The right hand side of (1) is given by $\mathbf{G} = (-\sqrt{i}\mathbf{n} \wedge \mathbf{g}, \mathbf{g}/\sqrt{i})^{\text{T}}$ where $\mathbf{g}$ is the right-hand side of the IBC. System (1) is discretized using a triangulation $\mathcal{T}_h$ of Γ and the Nédélec or RWG edge-based finite elements. The discrete system can be solved using a GMRES iterative solver; alternatively, a quasi-diagonal system can be derived from (1), with primal unknowns $\mathbf{U}' = (u_1 + u_2, u_1 - u_2)^{\text{T}}$, which can be solved with a relaxed block Jacobi method, where $\mathcal{N}_R$ constitutes the off-diagonal block. The block matrices are then inverted using two imbricated conjugate gradients. Note that in the particular case $R = 0$, the system is simply solved with classical, well conditioned, conjugate gradients [8].

3. Microlocal discretization

We assume that the obstacle Ω is a bounded open *convex* domain in $\mathbb{R}^3$ of boundary Γ, illuminated by an incident *plane* wave. Under these assumptions, it is possible to obtain an approximation of the phase of the solution of Maxwell's equations, with error estimates, which can be written

$$\phi(\mathbf{x}) = k\phi_0(\mathbf{x}) + O(k^{1/3}) = k\hat{\mathbf{k}} \cdot \mathbf{x} + O(k^{1/3}), \tag{2}$$

where $\hat{\mathbf{k}}$ is the direction of the incident wave. The authors of the microlocal discretization we consider [1] have used this approximation to propose a new discretization scheme of the integral equations. It is based on the consideration of a new unknown $\tilde{\mathbf{J}}$ such that $\mathbf{J} = \tilde{\mathbf{J}}\,\mathrm{e}^{\mathrm{i}k\phi_0}$, where $\mathbf{J}$ and $\tilde{\mathbf{J}}$ are currents on Γ.

Let $\Gamma_h = \bigcup_{K \in \mathcal{T}_h} F_K(K) = \bigcup_{K \in \mathcal{T}_h} \widetilde{K}_h$, where $K \in \mathcal{T}_h$ are plane triangles, be the approximate surface (we consider here the notations of [2]). For a one degree finite element approximation, $K \equiv \widetilde{K}_h$. The new basis functions on Γ_h are $\tilde{\varphi}_{|\widetilde{K}_h}(\mathbf{x}) = (1/\sqrt{\tilde{g}_h(\mathbf{x})}) \cdot \mathrm{e}^{\mathrm{i}k\phi_0(\mathbf{x})}\varphi^\alpha(\xi)\tilde{\mathbf{e}}_h^\alpha(\xi)$. $\tilde{\mathbf{e}}_h^\alpha(\xi)$ are vectors tangential to $\widetilde{K}_h$, $\tilde{g}_h$ is a metric function and φ^α are the components of the usual edge-based finite element basis function on the plane triangles $K \ni \xi$: $\varphi_{|K} = \varphi^\alpha \mathbf{f}_\alpha(K)$, $\mathbf{f}_\alpha(K)$ being vectors of an orthonormal frame of K.

Error estimates obtained in [1,9] for the microlocal discretization allow to discretize the unknown on a coarse mesh Γ_c with a number of elements $N_c \sim \mathrm{O}(k^{2/3})$ instead of $\mathrm{O}(k^2)$. However these error estimates indicate also that one should consider a high degree of approximation of the geometry: at least a degree $l = 7$ for the IBC. Since this condition is very difficult to achieve, the authors proposed to introduce a dual mesh to evaluate the integrals involved in the formulation. This dual mesh is denoted by Γ_f and has a number of elements $N_f \sim \mathrm{O}(k^2)$, as for a classical discretization. The fine mesh Γ_f is based on a refinement of the plane elements of $\mathcal{T}_c$, and a projection of the small elements on the surface Γ. The elements of Γ_f should be of degree $l = 2$ [9], but we have only implemented the case $l = 1$ (see the numerical results): for a point $\mathbf{x} \in \overline{K}_h$, a plane element of $\mathcal{T}_f$, the basis functions associated to $\mathcal{T}_c$ then write $\bar{\varphi}_{|\overline{K}_h}(\mathbf{x}) = 1/\sqrt{\overline{g}_h}\,\mathrm{e}^{\mathrm{i}k\phi_0(\mathbf{x})}\varphi^\alpha(\xi)\overline{\mathbf{e}}_h^\alpha$, where $\overline{g}_h$ and $\overline{\mathbf{e}}_h^\alpha$ are constant on $\overline{K}_h$.

We denote by $\overline{K}_{ii_0} \in \mathcal{T}_f$, $i_0 = 1, \ldots, \mathcal{N}_i$, the i_0-th "sub-element" of $K_i \in \mathcal{T}_c$, and $\mathbf{x}_{ii_0}$ a quadrature point on $\overline{K}_{ii_0}$. The matrix $\mathcal{M}^{\Re}$ coming from discretization of operator T_r or K_r, or $T_r \pm K_r$ in the quasi-diagonal system case, is evaluated as follows:

$$\mathcal{M}_{ij}^{\Re} = \sum_{i_0=1}^{\mathcal{N}_i} \sum_{j_0=1}^{\mathcal{N}_j} \alpha_{jj_0}\overline{\alpha_{ii_0}}\,\overline{\varphi}_j(\mathbf{x}_{jj_0}) \cdot \overline{\varphi_i(\mathbf{x}_{ii_0})}G_r(\mathbf{x}_{ii_0}, \mathbf{x}_{jj_0}), \tag{3}$$

where i, j are two edges of Γ_c. The Matrix $\mathcal{M}^{\Im}$ with the imaginary part of the Green function is evaluated in the same way. The cost of evaluation of $\mathcal{M}^{\Re}$ (resp. $\mathcal{M}^{\Im}$) is of order $\mathrm{O}(k^2)$, due to the sums in (3).

4. Coupling with the fast multipole method

To reduce the cost of the matrix calculation involved by the microlocal discretization, the authors Abboud et al. [1] suggested the use of the stationary phase theory; however the numerical approach of that theory implies difficulties not yet solved in 3-D, to our knowledge. We suggest then an original use of the multilevel fast multipole method (MLFMM) to speed up the calculation of the matrices without the stationary phase theory. Through clusters of elements of the mesh, the MLFMM [7] uncouples the interaction between two points of Γ_f using the Gegenbauer series and an integral around the unit sphere S^2. Let y_m, j_m be the spherical Neumann and Bessel functions and P_m the Legendre polynomial. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be two points of Gauss quadrature in Γ_f; $\mathbf{O}_1$, $\mathbf{O}_2$ the centers of the two clusters C_1 and C_2 containing respectively $\mathbf{x}_1$ and $\mathbf{x}_2$. Then, $\mathbf{x}_1 - \mathbf{x}_2 = \mathbf{r}_0 + \mathbf{r}$ where $\mathbf{r}_0 = \mathbf{O}_1 - \mathbf{O}_2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{r}_i = \mathbf{x}_i - \mathbf{O}_i$. With $|\mathbf{r}_0| > |\mathbf{r}|$, the multipole approximation for the real part of the integral kernel of (1) is given by

$$\frac{\cos(k|\mathbf{x}_1 - \mathbf{x}_2|)}{4\pi|\mathbf{x}_1 - \mathbf{x}_2|} \approx -\frac{k}{(4\pi)^2}\sum_{p=1}^{P}\omega_p\,\mathrm{e}^{\mathrm{i}k\langle\hat{\mathbf{s}}_p, \mathbf{r}_1\rangle}T_{l,\mathbf{r}_0}^{\Re}(\hat{\mathbf{s}}_p)\,\mathrm{e}^{-\mathrm{i}k\langle\hat{\mathbf{s}}_p, \mathbf{r}_2\rangle}$$

$$\text{with } T_{l,\mathbf{r}_0}^{\Re}(\hat{\mathbf{s}}) = \sum_{m=0}^{l}(2m+1)\,\mathrm{i}^m y_m\big(k|\mathbf{r}_0|\big)P_m\big(\cos(\hat{\mathbf{s}}, \mathbf{r}_0)\big), \tag{4}$$

where $l \approx kd + c(kd)^{1/3}$, d is the diameter of the multipole boxes, c is a constant which depends only on the desired accuracy and $P = (2l + 1)(l + 1)$ is the number of quadrature points on the unit sphere. The MLFMM

needs also the consideration of an octree. It implies several levels of clusters C. A classical MLFMM uses all the levels of the octree, from level three to the fine level. For our use, we will see that we should not consider levels that are coarser than the level lev_c whose number of clusters is $\mathrm{O}(N_c)$. Let $\overline{V}(C)$ be defined for each cluster C of each level lev by $\overline{V}(C) = \{\widetilde{C} \mid \widetilde{C} \text{ far from } C \text{ and } \widetilde{C}_a \text{ close to } C_a\}$ if $lev > lev_c$ and $\overline{V}(C) = \{\widetilde{C} \mid \widetilde{C} \text{ far from } C\}$ if $lev = lev_c$, where C_a is the antecedent of C in the octree. Let $\overline{K}_i$ be the set $\{\mathbf{x}_{ii_0}, i_0 = 1, \ldots, \mathcal{N}_i\}$. Considering $\mathcal{M}^{\mathfrak{R}}$ given by (3), the multipole approximation of the far interactions, $\mathcal{M}_{\mathrm{ap}}^{\mathfrak{R},\mathrm{f}}$, corresponding to the couples of points that meet the assumption $|\mathbf{r}_0| > |\mathbf{r}|$ in (4), is given by the following algorithm (the same process can be used for $\mathcal{M}^{\mathfrak{I}}$):

- Step 1: Transfer functions: For each considered level, $\forall(\mathbf{O}_C - \mathbf{O}_{\widetilde{C}})$, $\widetilde{C} \in \overline{V}(C)$, $\forall p \in \{1, \ldots, P\}$

$$T_{l,\mathbf{O}_C-\mathbf{O}_{\widetilde{C}}}^{\mathfrak{R}}(\hat{\mathbf{s}}_p) = \sum_{m=0}^{l} (2m+1)\,\mathrm{i}^m\, y_m\big(k|\mathbf{O}_C - \mathbf{O}_{\widetilde{C}}|\big) P_m\big(\cos(\hat{\mathbf{s}}_p, \mathbf{O}_C - \mathbf{O}_{\widetilde{C}})\big). \tag{5}$$

- Step 2: Local translation (radiation functions): For the finest level, $\forall i \in \{1, \ldots, N_c\}$, $\forall C$ FMM cluster such that $\overline{K}_i \cap C \neq \emptyset$, $\forall p \in \{1, \ldots, P\}$

$$\mathbf{F}_{iC}(\hat{\mathbf{s}}_p) = \sum_{i_0/\mathbf{x}_{ii_0}\in C} \alpha_{ii_0} \overline{\varphi}_i(\mathbf{x}_{ii_0})\,\mathrm{e}^{\mathrm{i}k\langle\hat{\mathbf{s}}_p, \mathbf{O}_C-\mathbf{x}_{ii_0}\rangle}. \tag{6}$$

The radiation functions at the upper levels are obtained by interpolation [5].
- Step 3: Matrix approximation: For each considered level, $\forall i, j \in \{1, \ldots, N_c\}$

$$\big(\mathcal{M}_{\mathrm{ap}}^{\mathfrak{R},\mathrm{f}}\big)_{ij} \longleftarrow \big(\mathcal{M}_{\mathrm{ap}}^{\mathfrak{R},\mathrm{f}}\big)_{ij} + \frac{-k}{(4\pi)^2}\sum_{p=1}^{P}\omega_p \sum_{C/\overline{K}_i\cap C\neq\emptyset} \overline{\mathbf{F}_{iC}(\hat{\mathbf{s}}_p)} \sum_{\substack{\widetilde{C}/\overline{K}_j\cap\widetilde{C}\neq\emptyset \\ \widetilde{C}\in\overline{V}(C)}} T_{l,\mathbf{O}_C-\mathbf{O}_{\widetilde{C}}}^{\mathfrak{R}}(\hat{\mathbf{s}}_p)\,\mathbf{F}_{j\widetilde{C}}(\hat{\mathbf{s}}_p). \tag{7}$$

Note that in our original use of the MLFMM, there is no descent in the octree.

A matricial element $\mathcal{M}_{ij}^{\mathfrak{R}}$ corresponds to an interaction between two objects $\overline{K}_i$ and $\overline{K}_j$. From an intuitive point of vue, it appears no use considering FMM clusters larger than the triangles K_i and K_j. Moreover, regarding the complexity of the different steps in detail, we clearly have to consider only the clusters smaller than the coarse triangles of Γ_c. Thus, we worked out a suitable multilevel fast multipole algorithm considering a reduced number of levels. Theoretically, an iterative solution using that new method has a memory cost of order $\mathrm{O}(N_f)$ and uses a CPU time of order $\mathrm{O}(N_f^{4/3}\ln N_f + N_{\mathrm{iter}}N_f^{2/3})$ where the term $\mathrm{O}(N_f^{4/3}\ln N_f)$ does not concern the CPU time of the iterative solution. Numerical results show the efficiency of the new algorithm in comparison with a classical use of the MLFMM.

5. Numerical results

We give now numerical results obtained for a sphere. The first result is given for the perfect conductor condition with a sphere of size $D = 26\lambda$ and the frequency $f = 3.2$ GHz where λ is the wavelength. The second result is obtained for the perfectly absorbing Léontovitch condition $Z = 1$ with $D = 38\lambda$ and $f = 4.7$ GHz. For both of them, the solution was based on a mesh with an average edge length about 1.6λ instead of the classical $\lambda/10$. Thus, the case using a fine mesh with $N_f = 327\,680$ triangles for $f = 3.2$ GHz (resp. $N_f = 737\,280$ for $f = 4.7$ GHz, more than one million degrees of freedom for a classical solution), is solved discretizing the unknown on a coarse mesh with $N_c = 1280$ triangles (resp. $N_c = 2880$). The figures show the bistatic Radar Cross Section (RCS) obtained with the new method denoted by FMD, in comparison with the exact solution given by the Mie series. Computing on a EV68 processor of a Compaq cluster ES40, for $f = 3.2$ GHz (resp. $f = 4.7$ GHz), the new method requires about 13 hours (resp. 30 hours) instead of about 35 hours (resp. 50 hours) for our implementation of MLFMM. For both cases, the solution is based on two imbricated conjugate gradients with a relative residual of
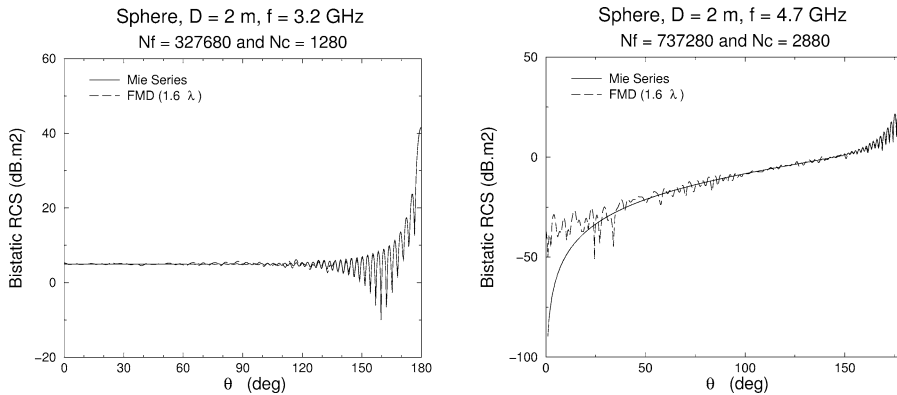
Fig. 1. Bistatic RCS.

Fig. 1. SER bistatique.

10^{-4}. For these cases, the new method is more efficient than a classical use of the MLFMM. For the Léontovitch condition $Z = 1$, the exact backward scattering amplitude vanishes which requires a high degree of accuracy. The comparison with the exact solution is then less good in this case. However, results should be more accurate with an approximation of the geometry of degree two, as prescribed by error estimates [1,9].

6. Conclusion

The method we have developed couples two kinds of methods in order to speed up the solution of integral equations. Firstly, a microlocal discretization enables us to reduce the size of the systems by approximating the phase function of the unknown. Secondly, an original use of the MLFMM enables us to speed up the calculation of the matrix of the new system. Such a combination has resulted in a new efficient method that has been performed within a new integral formulation with good properties for iterative solutions.

Now, we plan to work on a high degree approximation of the geometry for the absorbing boundary condition, on optimizations of the multilevel algorithm, and on an improvement of the approximation of the phase for non-convex objects.

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